Pensieve header: Analysis of k=2 invariants in QU: Aiming to find the relationship with OP2.

```
In[•]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\Projects\\SL2Portfolio"];
        << KnotTheory`
        << "SL2PortfolioProgram.m"
```

... **ParentDirectory**: Argument File should be a positive machine–size integer, a nonempty string, or a File specification.

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... **ToFileName**: String or list of strings expected at position 1 in ToFileName[{File, WikiLink, mathematica}].

... **ToFileName**: String or list of strings expected at position 1 in ToFileName[{File, QuantumGroups}].

```
Loading KnotTheory` version of January 20, 2015, 10:42:19.1122.
Read more at http://katlas.org/wiki/KnotTheory.
```

```
In[•]:= OverbayP2Data = Get["C:\\drorbn\\AcademicPensieve\\People\\Overbay\\OverbayP2Data.m"];
        OP2[K_Knot] := K /. OverbayP2Data /. T → T^(1/2);
```

```
In[•]:= {Length[OverbayP2Data], Last[OverbayP2Data]}
```

$$Out[•]= \left\{35, \text{Knot}[8, 21] \to 2670 + \frac{1}{T^{14}} + \frac{4}{T^{12}} - \frac{60}{T^{10}} + \frac{276}{T^8} - \right.$$
$$\left. \frac{775}{T^6} + \frac{1550}{T^4} - \frac{2331}{T^2} - 2331\,T^2 + 1550\,T^4 - 775\,T^6 + 276\,T^8 - 60\,T^{10} + 4\,T^{12} + T^{14}\right\}$$

```
In[•]:= $p = 5; $k = 2; $U = QU;
```

```
In[•]:= SCθ[p_] := Collect[ Cθ@𝕆_CU[{y, a, x}, p] /. {CU → Times, γ | ℏ → 1}, ε, Simplify];
        SQθ[p_] := Collect[ Qθ@𝕆_QU[{y, a, x}, p] /. {QU → Times, γ | ℏ → 1}, ε, Simplify];
```

```
In[•]:= 𝔼[L_, Q_, P_]_$k_ := 𝔼[L, Q, Series[Normal@P, {ε, 0, $k}]];
        𝔼_d→r_[L_, Q_, P_]_$k_ := 𝔼_d→r @@ 𝔼[L, Q, P]_$k;
        𝔼3@𝔼[ω_, L_, Q_, Ps_] := CF /@ 𝔼[L, ω^(-1) Q, ω^(-1) (ω^(-4) ε)^(-1+Range@Length@Ps) . Ps]_$k;
        𝔼4@𝔼[L_, Q_, P_] := Module[
            {ω = Normal[P]^(-1) /. ε → 0, Ps = CoefficientList[P, ε]},
            CF /@ 𝔼[ω, L, ω Q, ω^(-3+4 Range@Length@Ps) Ps]];
        𝔼3@𝔼_sp___[as___] := 𝔼3@𝔼[as] /. 𝔼 → 𝔼_sp;
        𝔼4@𝔼_sp___[as___] := 𝔼4@𝔼[as] /. 𝔼 → 𝔼_sp;
```

```
In[•]:= Clear[QP, ω];
        QP[Knot[n_, k_]] := QP[Knot[n, k]] = Collect[Module[{fname},
             fname = "../SL2Invariant/k=2/Data/" <> ToString[n] <> "_" <> ToString[k] <> ".m";
             Collect[𝔼3[Get[fname][[2, 2]]][[3]] // Normal, ε, Simplify]
            ], ε, CF];
        ω[K_Knot] := ω[K] = Factor[ (QP@K /. ε → 0)^(-1)];
        c_{k_,d_}[K_Knot] :=
          Factor[SeriesCoefficient[QP[K], {y, 0, 0}, {ε, 0, k}, {a, 0, d}] ω[K]^(1+2 k-d)]
```

In[◦]:=
```
H[p_] := If[TrueQ@Simplify[p == (p /. T → 1/T)],
    σ @@ CoefficientList[Expand@Together[p] /. T^n_ /; n < 0 :> 0, T], p];
H[p_] := If[TrueQ@Simplify[p == (p /. T → 1/T)],
    Style[Expand@Together[p] /. T^n_ /; n < 0 :> 0, Background → Yellow], p];
H[p_] := If[TrueQ@Simplify[p == (p /. T → 1/T)], Style[p, Background → Yellow], p];
H[l_List] := H /@ l;
```

In[◦]:=
```
MatrixForm[Table[
  H /@ Factor /@ {ω[K], c_{0,0}[K], c_{1,0}[K], c_{1,1}[K], c_{2,0}[K], c_{2,1}[K], c_{2,2}[K] OP2[K]},
  {K, AllKnots[{3, 7}]}
 ]]
```

Out[◦]//MatrixForm=

$$
\begin{pmatrix}
\frac{1-T+T^2}{T} & 1 & \frac{(-1+T)\,(2-T+T^2)}{T^2} & \frac{2\,(-1+T)\,(1+T)}{T} \\[4pt]
-\frac{1-3\,T+T^2}{T} & 1 & \frac{(-1+T)\,(1+T)\,(1-3\,T+T^2)}{T^2} & -\frac{2\,(-1+T)\,(1+T)}{T} \\[4pt]
\frac{1-T+T^2-T^3+T^4}{T^2} & 1 & \frac{(-1+T)\,(4-3\,T+5\,T^2-3\,T^3+3\,T^4-T^5+T^6)}{T^4} & \frac{2\,(-1+T)\,(1+T)\,(2-T+2\,T^2)}{T^2} \\[4pt]
\frac{2-3\,T+2\,T^2}{T} & 1 & -\frac{(-1+T)\,(-9+11\,T-7\,T^2+T^3)}{T^2} & \frac{4\,(-1+T)\,(1+T)}{T} \\[4pt]
-\frac{(-2+T)\,(-1+2\,T)}{T} & 1 & \frac{(-1+T)\,(5-11\,T-T^2+3\,T^3)}{T^2} & -\frac{4\,(-1+T)\,(1+T)}{T} \\[4pt]
-\frac{1-3\,T+3\,T^2-3\,T^3+T^4}{T^2} & 1 & \frac{(-1+T)\,(3-12\,T+16\,T^2-12\,T^3+4\,T^4-2\,T^6+T^7)}{T^4} & -\frac{2\,(-1+T)\,(1+T)\,(2-3\,T+2\,T^2)}{T^2} \\[4pt]
\frac{1-3\,T+5\,T^2-3\,T^3+T^4}{T^2} & 1 & \frac{(-1+T)\,(1+T)\,(2-3\,T+2\,T^2)\,(1-3\,T+5\,T^2-3\,T^3+T^4)}{T^4} & \frac{2\,(-1+T)\,(1+T)\,(2-3\,T+2\,T^2)}{T^2} \\[4pt]
\frac{1-T+T^2-T^3+T^4-T^5+T^6}{T^3} & 1 & \frac{(-1+T)\,(6-5\,T+9\,T^2-7\,T^3+9\,T^4-6\,T^5+6\,T^6-3\,T^7+3\,T^8-T^9+T^{10})}{T^6} & \frac{2\,(-1+T)\,(1+T)\,(3-2\,T+4\,T^2-2\,T^3+3\,T^4)}{T^3} \quad \frac{(-1+T)\,(-36}{} \\[4pt]
\frac{3-5\,T+3\,T^2}{T} & 1 & -\frac{(-1+T)\,(-23+36\,T-24\,T^2+5\,T^3)}{T^2} & \frac{6\,(-1+T)\,(1+T)}{T} \\[4pt]
\frac{2-3\,T+3\,T^2-3\,T^3+2\,T^4}{T^2} & 1 & \frac{(-1+T)\,(-1+7\,T-13\,T^2+24\,T^3-32\,T^4+35\,T^5-27\,T^6+17\,T^7)}{T^4} & \frac{2\,(-1+T)\,(1+T)\,(4-3\,T+4\,T^2)}{T^2} \\[4pt]
\frac{4-7\,T+4\,T^2}{T} & 1 & \frac{4\,(-1+T)\,(-2+11\,T-17\,T^2+10\,T^3)}{T^2} & \frac{8\,(-1+T)\,(1+T)}{T} \\[4pt]
\frac{2-4\,T+5\,T^2-4\,T^3+2\,T^4}{T^2} & 1 & -\frac{(-1+T)\,(-17+41\,T-65\,T^2+65\,T^3-49\,T^4+25\,T^5-9\,T^6+T^7)}{T^4} & \frac{8\,(-1+T)\,(1+T)\,(1-T+T^2)}{T^2} \quad (-1+ \\[4pt]
-\frac{1-5\,T+7\,T^2-5\,T^3+T^4}{T^2} & 1 & \frac{(-1+T)\,(3-22\,T+53\,T^2-53\,T^3+25\,T^4-T^5-4\,T^6+T^7)}{T^4} & -\frac{2\,(-2+T)\,(-1+T)\,(1+T)\,(-1+2\,T)}{T^2} \\[4pt]
\frac{1-5\,T+9\,T^2-5\,T^3+T^4}{T^2} & 1 & \frac{(-1+T)\,(2-13\,T+27\,T^2-9\,T^3-31\,T^4+33\,T^5-13\,T^6+2\,T^7)}{T^4} & \frac{2\,(-2+T)\,(-1+T)\,(1+T)\,(-1+2\,T)}{T^2} \quad (-1
\end{pmatrix}
$$

In[◦]:=
```
p1[K_Knot] := p1[K] = Factor[ T (-c_{1,0}[K] + ω[K] T ∂_T ω[K]) / (T - 1)^2 ];

ρ1[K_Knot] := ρ1[K] = Factor[ (T - 1)^2 / T  p1[K] ];
```

In[◦]:=
```
p2[K_Knot] := p2[K] = CF[-2 c_{2,0}[K] + ω[K] c_{2,1}[K]];
```

In[◦]:=
```
p2[Knot[8, 21]]
```

Out[◦]=
$$
\frac{1}{T^8}\Big(3 - 28\,T + 49\,T^2 + 352\,T^3 - 2489\,T^4 + 8164\,T^5 - 17530\,T^6 + 27092\,T^7 - 31226\,T^8 + 27092\,T^9 - 17530\,T^{10} + 8164\,T^{11} - 2489\,T^{12} + 352\,T^{13} + 49\,T^{14} - 28\,T^{15} + 3\,T^{16}\Big)
$$

```mathematica
In[*]:=  MyCollect[ℰ_, vs_List] := MyCollect[ℰ, vs, Identity];
         MyCollect[ℰ_, vs_List, simp_] :=
           Total[CoefficientRules[ℰ, vs] /. ((ps_ → c_) :> simp[c] Times @@ (vs^ps))];
         MyCollect[ℰs_List, vs_List] := MyCollect[#, vs] & /@ ℰs;
         MyCollect[ℰs_List, vs_List, simp_] := MyCollect[#, vs, simp] & /@ ℰs;
         MyCollect[sd_SeriesData, vs_List] := MapAt[MyCollect[#, vs] &, sd, 3];
         MyCollect[sd_SeriesData, vs_List, simp_] := MapAt[MyCollect[#, vs, simp] &, sd, 3];
```

```mathematica
In[*]:=  Monitor[Total[Table[
           Simplify[(c_{0,0}[K] == 1) ∧ (2 T ∂_T ω[K] == c_{1,1}[K]) ∧
             (c_{2,1}[K] == (2 (-1 + T) p1[K] ((1 + T) ω[K] - 3 (-1 + T) T ∂_T ω[K]))/T +

                2 ω[K] ((-1 + T)^2 ∂_T p1[K] + 2 T^2 (∂_T ω[K])^2 - T ω[K] (∂_T ω[K] + T ∂_{T,T} ω[K]))) ∧

             (c_{2,2}[K] == -2 T (-2 T (∂_T ω[K])^2 + ω[K] (∂_T ω[K] + T ∂_{T,T} ω[K])))],
           {K, AllKnots[{3, 10}]}]],
         K]
```

Out[*]= 249 True

```mathematica
In[*]:=  RecoveryFormula = ω^{-1} + ((-2 T ω dω)/(T - 1) x y + 2 T ω dω a + (ω T dω - (T - 1)^2/T p1)) ϵ/ω^3 +

           (2 T ω^2 (2 dω^2 T - dω ω - ddω T ω) a^2 + ((2 (-1 + T) p1 ((1 + T) ω - 3 (-1 + T) T dω))/T +

              2 ω ((-1 + T)^2 dp1 + 2 T^2 (dω)^2 - T ω (dω + T ddω))) ω a +

              (T ω^2 (4 dω^2 (-1 + T) T - dω (-3 + T) ω - 2 ddω (-1 + T) T ω))/(-1 + T)^3 x^2 y^2 -

              (2 ω (-3 dω p1 (-1 + T) T + dp1 (-1 + T) T ω + p1 (1 + T) ω))/T x y +

              (4 T ω^2 (2 dω^2 (1 - T) T - dω ω - ddω (1 - T) T ω))/(-1 + T)^2 a x y - (p2/2 + ω (3 dω p1 (-1 + T)^2 -

              (p1 (-1 + T^2) + T (dp1 (-1 + T)^2 + 2 dω^2 T^2)) ω)/T + T (dω + ddω T) ω^2))) ϵ^2/ω^5;
```

```
Monitor[MatrixForm@Table[
    H@Simplify[{ω, p1, p2, (T² ω dω)/(1 - T²), (-p1 (-1 + T)² + dω T² ω)/T,
        r = -QP[K] + RecoveryFormula}
      /. {ω → ω[K], dω → ∂_T ω[K], ddω → ∂_{T,T} ω[K], p1 → p1[K], dp1 → ∂_T p1[K], p2 → p2[K]}],
    {K, AllKnots[{3, 6}]}],
  K]
```

Out[ ]//MatrixForm=

$$
\begin{pmatrix}
-1 + \frac{1}{T} + T & \frac{1}{T} + T & -38 + \frac{3}{T^3} - \frac{1}{?} \\
3 - \frac{1}{T} - T & 0 & -110 + \frac{1}{T^4} - \frac{3}{T^3} - ? \\
1 + \frac{1}{T^2} - \frac{1}{T} - T + T^2 & \frac{(1+T^2)(2+T^2+2T^4)}{T^3} & -510 + \frac{5}{T^7} - \frac{20}{T^6} + \frac{55}{T^5} - \frac{120}{T^4} + \frac{217}{T^3} - \frac{338}{T^2} + \frac{45?}{T} \\
-3 + \frac{2}{T} + 2T & -4 + \frac{5}{T} + 5T & -1362 - \frac{10}{T^4} + \frac{120}{T^3} - \frac{487}{T^2} \\
5 - \frac{2}{T} - 2T & -4 + \frac{1}{T} + T & -1598 + \frac{14}{T^4} - \frac{16}{T^3} - \frac{293}{T^2} + ? \\
-3 - \frac{1}{T^2} + \frac{3}{T} + 3T - T^2 & -4 + \frac{1}{T^3} - \frac{4}{T^2} + \frac{4}{T} + 4T - 4T^2 + T^3 & -6410 + \frac{3}{T^8} - \frac{21}{T^7} + \frac{49}{T^6} + \frac{15}{T^5} - \frac{433}{T^4} + \frac{1543}{T^3} - \frac{3431}{T^2} + \frac{5482}{T} \\
5 + \frac{1}{T^2} - \frac{3}{T} - 3T + T^2 & 0 & -8510 + \frac{4}{T^8} - \frac{33}{T^7} + \frac{121}{T^6} - \frac{203}{T^5} - \frac{111}{T^4} + \frac{1499}{T^3} - \frac{4210}{T^2} + \frac{7186}{T}
\end{pmatrix}
$$

In[ ]:=
```
Monitor[Union@Table[
    Simplify[r = -QP[K] + RecoveryFormula /.
      {ω → ω[K], dω → ∂_T ω[K], ddω → ∂_{T,T} ω[K], p1 → p1[K], dp1 → ∂_T p1[K], p2 → p2[K]}],
    {K, AllKnots[{3, 10}]}],
  K]
```

Out[ ]= {0}

In[ ]:= `MyCollect[Log[(ω RecoveryFormula /. ϵ → ω² ϵ) + O[ϵ]³], {a, x, y}, FullSimplify]`

Out[ ]=

$$
\left(-\frac{p1 (-1 + T)^2}{T} + dω\, T\, ω + 2\, a\, dω\, T\, ω - \frac{2\, dω\, T\, x\, y\, ω}{-1 + T} \right) \epsilon + \left(-2\, a^2\, T\, ω^2 \left(-dω^2\, T + dω\, ω + ddω\, T\, ω \right) + \right.
$$

$$
\frac{T\, x^2\, y^2\, ω^2 \left(2\, dω^2 (-1 + T)\, T - dω (-3 + T)\, ω - 2\, ddω (-1 + T)\, T\, ω \right)}{(-1 + T)^3} -
$$

$$
\frac{4\, a\, T\, x\, y\, ω^2 \left(dω^2 (-1 + T)\, T + dω\, ω - ddω (-1 + T)\, T\, ω \right)}{(-1 + T)^2} +
$$

$$
2\, x\, y\, ω \left(2\, dω\, p1 (-1 + T) + \frac{dω^2\, T^2\, ω}{-1 + T} - \frac{(dp1 (-1 + T)\, T + p1 (1 + T))\, ω}{T} \right) +
$$

$$
\frac{2\, a\, ω \left(-2\, dω\, p1 (-1 + T)^2\, T + p1 (-1 + T^2)\, ω + T \left(dp1 (-1 + T)^2 + dω^2\, T^2 \right) ω - T^2 (dω + ddω\, T)\, ω^2 \right)}{T} +
$$

$$
\frac{1}{2\, T^2} \left(-p1^2 (-1 + T)^4 - 2\, p1 (-1 + T)\, T\, ω \left(2\, dω (-1 + T)\, T - (1 + T)\, ω \right) + \right.
$$

$$
\left. \left. T^2 \left(-p2 + ω^2 \left(2\, dp1 (-1 + T)^2 + T \left(3\, dω^2\, T - 2 (dω + ddω\, T)\, ω \right) \right) \right) \right) \right) \epsilon^2 + O[\epsilon]^3
$$

C_{kij} is the coefficient of $\epsilon^k\, a^i (xy)^j$ in Log[(ω RecoveryFormula /. $\epsilon \to \omega^2 \epsilon$) + $O[\epsilon]^3$]; T_{kij} is its TeXForm:

```
In[•]:= T100 = ToString@TeXForm[(C100 = HoldForm[T ω dω - p1 (T - 1)^2 / T]) /. {dω → ω', p1 → p1}];

    T110 = ToString@TeXForm[(C110 = HoldForm[2 T ω dω]) /. {dω → ω', p1 → p1}];

    T101 = ToString@TeXForm[(C101 = HoldForm[2 T ω dω / (1 - T)]) /. {dω → ω', p1 → p1}];

    If[
      Simplify[SeriesCoefficient[Log[(ω RecoveryFormula /. ε → ω^2 ε) + O[ε]^3], 1] ==
        ReleaseHold[C100] + ReleaseHold[C110] a + ReleaseHold[C101] x y],
      ToString@StringReplace["\\[
        P^{(1)} =
          \\left(T100 \\right)
          + T110 a
          + T101 xy,
    \\]",
        {"T100" → T100, "T110" → T110, "T101" → T101}]]
```

```
Out[•]= \[
        P^{(1)} =
          \left(T \omega  \omega '-\frac{p_1 (T-1)^2}{T} \right)
          + 2 T \omega  \omega ' a
          + \frac{2 T \omega  \omega '}{1-T} xy,
    \]
```

In[•]:=
```
T₂₀₀ = ToString@
    TeXForm[(C₂₀₀ = HoldForm[ 1/(2 T²) (T² (ω² (2 dp1 (T - 1)² + T (3 dω² T - 2 (dω + ddω T) ω)) - p2) -
              p1² (T - 1)⁴ - 2 p1 (T - 1) T ω (2 dω (T - 1) T - (1 + T) ω))]) /.
      {dω → ω', ddω → ω'', p1 → p₁, dp1 → p₁', p2 → p₂}];

T₂₁₀ = ToString@TeXForm[(C₂₁₀ = HoldForm[

        2 ω (p1 (T² - 1) ω - 2 dω p1 (T - 1)² T + T (dp1 (T - 1)² + dω² T²) ω - T² (dω + ddω T) ω²)/T]) /.

      {dω → ω', ddω → ω'', p1 → p₁, dp1 → p₁', p2 → p₂}];
T₂₂₀ = ToString@TeXForm[(C₂₂₀ = HoldForm[2 T ω² (dω² T - dω ω - ddω T ω)]) /.
      {dω → ω', ddω → ω'', p1 → p₁, dp1 → p₁', p2 → p₂}];

T₂₀₁ = ToString@TeXForm[(C₂₀₁ = HoldForm[

        2 ω (2 dω p1 (T - 1) + (dω² T² ω)/(T - 1) - ((dp1 (T - 1) T + p1 (1 + T)) ω)/T)]) /.
      {dω → ω', ddω → ω'', p1 → p₁, dp1 → p₁', p2 → p₂}];
T₂₁₁ = ToString@TeXForm[(C₂₁₁ = HoldForm[(4 T ω² (ddω (T - 1) T ω - dω² (T - 1) T - dω ω))/(T - 1)²]) /.

      {dω → ω', ddω → ω'', p1 → p₁, dp1 → p₁', p2 → p₂}];
T₂₀₂ = ToString@TeXForm[(C₂₀₂ = HoldForm[

        (T ω² (2 dω² (T - 1) T - dω (T - 3) ω - 2 ddω (T - 1) T ω))/(T - 1)³]) /.
      {dω → ω', ddω → ω'', p1 → p₁, dp1 → p₁', p2 → p₂}];
If[
 Simplify[SeriesCoefficient[Log[(ω RecoveryFormula /. ε → ω² ε) + O[ε]³], 2] ==
    ReleaseHold[C₂₀₀] + ReleaseHold[C₂₁₀] a + ReleaseHold[C₂₂₀] a² +
     ReleaseHold[C₂₀₁] x y + ReleaseHold[C₂₁₁] a x y + ReleaseHold[C₂₀₂] x² y²],
  ToString@StringReplace["\\begin{multline*}
  P^{(2)} =
    T200 \\\\
    + T210 a \\\\
    + T220 a^2
    + T201 x y \\\\
    + T211 a x y
    + T202 x^2y^2.
\\end{multline*}",
    {"T200" → T₂₀₀, "T210" → T₂₁₀, "T220" → T₂₂₀, "T201" → T₂₀₁, "T211" → T₂₁₁, "T202" → T₂₀₂}]]
```

Out[•]=
```
\begin{multline*}
   P^{(2)} =
      \frac{T^2 \left(\omega ^2 \left(2 p_1' (T-1)^2+T
   \left(3 \left(\omega '\right)^2 T-2 \left(\omega '+\omega ''
   T\right) \omega \right)\right)-p_2\right)-p_1^2 (T-1)^4-2 p_1 (T-1)
   T \omega  \left(2 \omega ' (T-1) T-(1+T) \omega \right)}{2 T^2} \\
      + \frac{2 \omega  \left(p_1 \left(T^2-1\right) \omega -2 \omega '
   p_1 (T-1)^2 T+T \left(p_1' (T-1)^2+\left(\omega '\right)^2 T^2\right)
   \omega -T^2 \left(\omega '+\omega '' T\right) \omega ^2\right)}{T} a \\
      + 2 T \omega ^2 \left(\left(\omega '\right)^2
   T-\omega ' \omega -\omega '' T \omega \right) a^2
      + 2 \omega  \left(2 \omega ' p_1 (T-1)+\frac{\left(\omega '\right)^2 T^2 \omega
   }{T-1}-\frac{\left(p_1' (T-1) T+p_1 (1+T)\right) \omega }{T}\right) x y \\
      + \frac{4 T \omega ^2 \left(\omega '' (T-1) T \omega -\left(\omega
   '\right)^2 (T-1) T-\omega ' \omega \right)}{(T-1)^2} a x y
      + \frac{T \omega ^2 \left(2 \left(\omega '\right)^2 (T-1) T-\omega
   ' (T-3) \omega -2 \omega '' (T-1) T \omega \right)}{(T-1)^3} x^2y^2.
\end{multline*}
```

```mathematica
In[ ]:= MatrixForm[mat = Table[Module[{r, ω = ω[K], dω = ∂_T ω[K], ddω = ∂_{T,T} ω[K],
            dddω = ∂_{T,T,T} ω[K], p1 = p1[K], dp1 = ∂_T p1[K], ddp1 = ∂_{T,T} p1[K], p2 = p2[K]},
          Factor@{
            r = -QP[K] + ω^{-1} +
              ( (-2 T dω)/((T-1) ω²) x y + (2 T dω a)/ω² + (ω T dω - ((T-1)²/T) p1)/ω³ ) ε + ( (-2 T (-2 T dω² + ω (dω + T ddω)) a²)/ω³ +
              ( (2(-1+T) p1 ((1+T) ω - 3 (-1+T) T dω))/T + 2 ω ((-1+T)² dp1 + 2 T² (dω)² - T ω (dω + T ddω)) )/ω⁴ ) a -
              2 ( (-2 dω² T² + ((T-3) dω T ω)/(2 T-2) + ddω T² ω)/((T-1)² ω³) ) x² y² - ((T²-1)/((T+1) T/2 ω⁴)) (-3 dω p1 T +
                ((T+1) p1 ω)/(T-1) + dp1 T ω) x y + 4/((T-1) ω³) (-2 dω² T² - (dω T ω)/(T-1) + ddω T² ω) a x y) ε²;
            Coefficient[ω⁵ r, ε²],
            p2,
            ω² p1, ω p1, p1, ω p1 T dω,
            p1 T dω, T dp1, ω T dp1, ω² T dp1, T² ∂_{T,T}ρ1, ω T² ddp1, T² dω dp1,
            p1², p1 T dp1,
            ω⁴, ω³, ω², ω,
            ω³ T dω, ω² T dω, ω T dω, T dω,
            ω² (T dω)², ω (T dω)², (T dω)²,
            ω³ T² ddω, ω² T² ddω, ω T² ddω, T² ddω,
            ω T³ dddω, T³ dddω
          }],
          {K, AllKnots[{3, 8}]}
        ]];
      Dimensions[mat]
      MatrixRank[mat /. T → 10⁶]
      NullSpace[mat /. T → 10³]

Out[ ]= {35, 32}

Out[ ]= 29

Out[ ]= {{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -(2006006002/332334333),
        2006006002/332334333, 0, 0, 2008016012006/333332666667, 0, 0, 0, 0, 0, 0, -(334001/111111), 0, 1, 0},
       {-1, 1/2, -(999999/1000), 0, 0, 2994003/1000, 0, 0, 0, -(998001/1000), 0, 0, 0,
        0, 0, 0, 0, 0, 1, 0, 0, 0, -2, 0, 0, 1, 0, 0, 0, 0, 0},
       {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}}}
```

$In[\circ]:=$ $\left\{-1, \dfrac{1}{2}, -\dfrac{999\,999}{1000}, 0, 0, \dfrac{2\,994\,003}{1000}, 0, 0, 0, -\dfrac{998\,001}{1000},\right.$

$\left. 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, -2, 0, 0, 1, 0, 0, 0, 0, 0\right\}.\{$

$\quad$ r, p2,

$\quad \omega^2\,p1\,,\ \omega\,p1\,,\ p1\,,\ \omega\,p1\,T\,d\omega\,,\ p1\,T\,d\omega\,,\ T\,dp1\,,\ \omega\,T\,dp1\,,\ \omega^2\,T\,dp1\,,\ T^2\,\partial_{T,T}\rho1\,,\ \omega\,T^2\,ddp1\,,\ T^2\,d\omega\,dp1\,,$

$\quad p1^2\,,\ p1\,T\,dp1\,,$

$\quad \omega^4,\ \omega^3,\ \omega^2,\ \omega,$

$\quad \omega^3\,T\,d\omega\,,\ \omega^2\,T\,d\omega\,,\ \omega\,T\,d\omega\,,\ T\,d\omega\,,$

$\quad \omega^2\,\left(T\,d\omega\right)^2,\ \omega\,\left(T\,d\omega\right)^2,\ \left(T\,d\omega\right)^2,$

$\quad \omega^3\,T^2\,dd\omega\,,\ \omega^2\,T^2\,dd\omega\,,\ \omega\,T^2\,dd\omega\,,\ T^2\,dd\omega\,,$

$\quad \omega\,T^3\,ddd\omega\,,\ T^3\,ddd\omega$

$\quad \}$

$Out[\circ]=$ $\dfrac{p2}{2} - r + \dfrac{2\,994\,003\ d\omega\ p1\ T\ \omega}{1000} - \dfrac{999\,999\ p1\ \omega^2}{1000} - \dfrac{998\,001\ dp1\ T\ \omega^2}{1000} - 2\ d\omega^2\ T^2\ \omega^2 + d\omega\ T\ \omega^3 + dd\omega\ T^2\ \omega^3$

$In[\circ]:=$
```
MatrixForm[Table[
  H /@ Together /@
    Expand /@ {ω[K], p1[K], ρ1[K], -2 c_{2,0}[K] + ω[K] c_{2,1}[K], OP2[K], OP2[K] / ω[K]},
  {K, AllKnots[{3, 8}]}
  ]]
```

$Out[\circ]//MatrixForm=$

$\dfrac{1-T+T^2}{T}$	$\dfrac{1+T^2}{T}$	$\dfrac{1-2\,T+2\,T^2-2\,T^3}{T^2}$
$\dfrac{-1+3\,T-T^2}{T}$	0	0
$\dfrac{1-T+T^2-T^3+T^4}{T^2}$	$\dfrac{2+3\,T^2+3\,T^4+2\,T^6}{T^3}$	$\dfrac{2-4\,T+5\,T^2-6\,T^3+6\,T^4-6\,T^5+}{T^4}$
$\dfrac{2-3\,T+2\,T^2}{T}$	$\dfrac{5-4\,T+5\,T^2}{T}$	$\dfrac{5-14\,T+18\,T^2-14\,T}{T^2}$
$\dfrac{-2+5\,T-2\,T^2}{T}$	$\dfrac{1-4\,T+T^2}{T}$	$\dfrac{1-6\,T+10\,T^2-6\,T}{T^2}$
$\dfrac{-1+3\,T-3\,T^2+3\,T^3-T^4}{T^2}$	$\dfrac{1-4\,T+4\,T^2-4\,T^3+4\,T^4-4\,T^5+T^6}{T^3}$	$\dfrac{1-6\,T+13\,T^2-16\,T^3+16\,T^4-16}{T^4}$
$\dfrac{1-3\,T+5\,T^2-3\,T^3+T^4}{T^2}$	0	0
$\dfrac{1-T+T^2-T^3+T^4-T^5+T^6}{T^3}$	$\dfrac{3+5\,T^2+6\,T^4+6\,T^6+5\,T^8+3\,T^{10}}{T^5}$	$\dfrac{3-6\,T+8\,T^2-10\,T^3+11\,T^4-12\,T^5+12\,T^6-12\,T^7}{T^6}$
$\dfrac{3-5\,T+3\,T^2}{T}$	$\dfrac{2\,\left(7-8\,T+7\,T^2\right)}{T}$	$\dfrac{2\,\left(7-22\,T+30\,T^2-22\right)}{T^2}$
$\dfrac{2-3\,T+3\,T^2-3\,T^3+2\,T^4}{T^2}$	$\dfrac{-9+8\,T-16\,T^2+12\,T^3-16\,T^4+8\,T^5-9\,T^6}{T^3}$	$\dfrac{-9+26\,T-41\,T^2+52\,T^3-56\,T^4+52}{T^4}$
$\dfrac{4-7\,T+4\,T^2}{T}$	$-\dfrac{8\,\left(3-4\,T+3\,T^2\right)}{T}$	$-\dfrac{8\,\left(3-10\,T+14\,T^2-16\right)}{T^2}$
$\dfrac{2-4\,T+5\,T^2-4\,T^3+2\,T^4}{T^2}$	$\dfrac{9-16\,T+29\,T^2-28\,T^3+29\,T^4-16\,T^5+9\,T^6}{T^3}$	$\dfrac{9-34\,T+70\,T^2-102\,T^3+114\,T^4-102}{T^4}$
$\dfrac{-1+5\,T-7\,T^2+5\,T^3-T^4}{T^2}$	$\dfrac{1-8\,T+19\,T^2-20\,T^3+19\,T^4-8\,T^5+T^6}{T^3}$	$\dfrac{1-10\,T+36\,T^2-66\,T^3+78\,T^4-66}{T^4}$
$\dfrac{1-5\,T+9\,T^2-5\,T^3+T^4}{T^2}$	$\dfrac{-3+8\,T-3\,T^2}{T}$	$\dfrac{-3+14\,T-22\,T^2+14\,T}{T^2}$
$\dfrac{-3+7\,T-3\,T^2}{T}$	$\dfrac{5-16\,T+5\,T^2}{T}$	$\dfrac{5-26\,T+42\,T^2-26\,T}{T^2}$
$\dfrac{-1+3\,T-3\,T^2+3\,T^3-3\,T^4+3\,T^5-T^6}{T^3}$	$\dfrac{2-8\,T+10\,T^2-12\,T^3+13\,T^4-12\,T^5+13\,T^6-12\,T^7+10\,T^8-8\,T^9+2\,T^{10}}{T^5}$	$\dfrac{2-12\,T+28\,T^2-40\,T^3+47\,T^4-50\,T^5+50\,T^6-50\,T^7}{T^6}$
$\dfrac{-4+9\,T-4\,T^2}{T}$	0	0

$\frac{-2+5T-5T^2+5T^3-2T^4}{T^2}$	$\frac{3-8T+6T^2-4T^3+6T^4-8T^5+3T^6}{T^3}$	$\frac{3-14T+25T^2-24T^3+20T^4-24T\ldots}{T^4}$
$\frac{-1+3T-4T^2+5T^3-4T^4+3T^5-T^6}{T^3}$	$\frac{-2+8T-13T^2+20T^3-22T^4+24T^5-22T^6+20T^7-13T^8+8T^9-2T^{10}}{T^5}$	$\frac{-2+12T-31T^2+54T^3-75T^4+88T^5-92T^6+88T\ldots}{T^6}$
$\frac{-2+6T-7T^2+6T^3-2T^4}{T^2}$	$\frac{5-20T+28T^2-32T^3+28T^4-20T^5+5T^6}{T^3}$	$\frac{5-30T+73T^2-108T^3+120T^4-108\ldots}{T^4}$
$\frac{1-3T+5T^2-5T^3+5T^4-3T^5+T^6}{T^3}$	$\frac{-1+4T-10T^2+12T^3-13T^4+12T^5-13T^6+12T^7-10T^8+4T^9-T^{10}}{T^5}$	$\frac{-1+6T-19T^2+36T^3-47T^4+50T^5-50T^6+50T\ldots}{T^6}$
$\frac{2-6T+9T^2-6T^3+2T^4}{T^2}$	$\frac{-1+4T-12T^2+16T^3-12T^4+4T^5-T^6}{T^3}$	$\frac{-1+6T-21T^2+44T^3-56T^4+44\ldots}{T^4}$
$\frac{-1+3T-5T^2+7T^3-5T^4+3T^5-T^6}{T^3}$	0	0
$\frac{1-3T+6T^2-7T^3+6T^4-3T^5+T^6}{T^3}$	$\frac{-1+4T-11T^2+16T^3-21T^4+20T^5-21T^6+16T^7-11T^8+4T^9-T^{10}}{T^5}$	$\frac{-1+6T-20T^2+42T^3-64T^4+78T^5-82T^6+78T\ldots}{T^6}$
$\frac{-2+7T-9T^2+7T^3-2T^4}{T^2}$	$\frac{5-24T+39T^2-44T^3+39T^4-24T^5+5T^6}{T^3}$	$\frac{5-34T+92T^2-146T^3+166T^4-146\ldots}{T^4}$
$\frac{1-7T+13T^2-7T^3+T^4}{T^2}$	0	0
$\frac{2-7T+11T^2-7T^3+2T^4}{T^2}$	$\frac{-1+4T-14T^2+20T^3-14T^4+4T^5-T^6}{T^3}$	$\frac{-1+6T-23T^2+52T^3-68T^4+52\ldots}{T^4}$
$\frac{-2+8T-11T^2+8T^3-2T^4}{T^2}$	$\frac{5-28T+57T^2-68T^3+57T^4-28T^5+5T^6}{T^3}$	$\frac{5-38T+118T^2-210T^3+250T^4-210\ldots}{T^4}$
$\frac{3-8T+11T^2-8T^3+3T^4}{T^2}$	$\frac{21-64T+120T^2-140T^3+120T^4-64T^5+21T^6}{T^3}$	$\frac{21-106T+269T^2-444T^3+520T^4-444\ldots}{T^4}$
$\frac{1-4T+8T^2-9T^3+8T^4-4T^5+T^6}{T^3}$	$\frac{1-6T+17T^2-28T^3+35T^4-36T^5+35T^6-28T^7+17T^8-6T^9+T^{10}}{T^5}$	$\frac{1-8T+30T^2-68T^3+108T^4-134T^5+142T^6-134\ldots}{T^6}$
$\frac{-1+4T-8T^2+11T^3-8T^4+4T^5-T^6}{T^3}$	0	0
$\frac{-1+5T-10T^2+13T^3-10T^4+5T^5-T^6}{T^3}$	0	0
$\frac{1-T+T^3-T^5+T^6}{T^3}$	$\frac{-3-4T^3-3T^4-3T^6-4T^7-3T^{10}}{T^5}$	$\frac{-3+6T-3T^2-4T^3+5T^4+2T^5-6T^6+2T^7+\ldots}{T^6}$
$\frac{1-2T+3T^2-2T^3+T^4}{T^2}$	$\frac{4\,(1-T+T^2)}{T}$	$\frac{4\,(1-3T+4T^2-3T\ldots)}{T^2}$
$\frac{-1+4T-5T^2+4T^3-T^4}{T^2}$	$\frac{1-8T+16T^2-20T^3+16T^4-8T^5+T^6}{T^3}$	$\frac{1-10T+33T^2-60T^3+72T^4-60\ldots}{T^4}$

```
In[ ]:= MatrixForm[Table[
    Together /@ Expand /@
      {ω[K], ∂_T ω[K], p1[K], ρ1[K], -2 c_{2,0}[K] + ω[K] c_{2,1}[K], OP2[K], OP2[K] / ω[K]},
    {K, AllKnots[{3, 8}]}
  ] /. T → 1]
```

Out[ ]//MatrixForm=

$$
\begin{pmatrix}
1 & 0 & 2 & 0 & -4 & 1 & 1 \\
1 & 0 & 0 & 0 & 4 & -1 & -1 \\
1 & 0 & 10 & 0 & -12 & 3 & 3 \\
1 & 0 & 6 & 0 & -8 & 2 & 2 \\
1 & 0 & -2 & 0 & 8 & -2 & -2 \\
1 & 0 & -2 & 0 & 4 & -1 & -1 \\
1 & 0 & 0 & 0 & -4 & 1 & 1 \\
1 & 0 & 28 & 0 & -24 & 6 & 6 \\
1 & 0 & 12 & 0 & -12 & 3 & 3 \\
1 & 0 & -22 & 0 & -20 & 5 & 5 \\
1 & 0 & -16 & 0 & -16 & 4 & 4 \\
1 & 0 & 16 & 0 & -16 & 4 & 4 \\
1 & 0 & 4 & 0 & -4 & 1 & 1 \\
1 & 0 & 2 & 0 & 4 & -1 & -1 \\
1 & 0 & -6 & 0 & 12 & -3 & -3 \\
1 & 0 & -2 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 16 & 4 & 4 \\
1 & 0 & -2 & 0 & 12 & -3 & -3 \\
1 & 0 & 6 & 0 & 4 & -1 & -1 \\
1 & 0 & -6 & 0 & 8 & -2 & -2 \\
1 & 0 & -4 & 0 & -8 & 2 & 2 \\
1 & 0 & -2 & 0 & -8 & 2 & 2 \\
1 & 0 & 0 & 0 & 8 & -2 & -2 \\
1 & 0 & -6 & 0 & -12 & 3 & 3 \\
1 & 0 & -4 & 0 & 4 & -1 & -1 \\
1 & 0 & 0 & 0 & 12 & -3 & -3 \\
1 & 0 & -2 & 0 & -4 & 1 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 14 & 0 & -16 & 4 & 4 \\
1 & 0 & 2 & 0 & -4 & 1 & 1 \\
1 & 0 & 0 & 0 & 4 & -1 & -1 \\
1 & 0 & 0 & 0 & -4 & 1 & 1 \\
1 & 0 & -20 & 0 & -20 & 5 & 5 \\
1 & 0 & 4 & 0 & -8 & 2 & 2 \\
1 & 0 & -2 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

$In[\circ]:=$ `MatrixForm[mat = Table[`
  `Factor@{`
    `OP2[K], OP2[K] /. T → -T, c_{2,1}[K], c_{2,2}[K], -2 c_{2,0}[K] + ω[K] c_{2,1}[K],`
    `ω[K]^2 ρ1[K], ω[K] ρ1[K], ρ1[K], ω[K] ρ1[K] T ∂_T ω[K], ρ1[K] T ∂_T ω[K], T ∂_T ρ1[K],`
    `ω[K] T ∂_T ρ1[K], ω[K]^2 T ∂_T ρ1[K], T^2 ∂_{T,T} ρ1[K], ω[K] T^2 ∂_{T,T} ρ1[K], T^2 ∂_{T,T} (ω[K] ρ1[K]),`
    `ρ1[K]^2, ρ1[K] T ∂_T ρ1[K],`
    `ω[K]^4, ω[K]^3, ω[K]^2, ω[K],`
    `ω[K]^3 T ∂_T ω[K], ω[K]^2 T ∂_T ω[K], ω[K] T ∂_T ω[K], T ∂_T ω[K],`
    `ω[K]^2 (T ∂_T ω[K])^2, ω[K] (T ∂_T ω[K])^2, (T ∂_T ω[K])^2,`
    `ω[K]^3 T^2 ∂_{T,T} ω[K], ω[K]^2 T^2 ∂_{T,T} ω[K], ω[K] T^2 ∂_{T,T} ω[K], T^2 ∂_{T,T} ω[K],`
    `ω[K] T^3 ∂_{T,T,T} ω[K], T^3 ∂_{T,T,T} ω[K]`
    `},`
  `{K, AllKnots[{3, 8}]}`
  `]];`
`Dimensions[mat]`
`MatrixRank[mat /. T → 10^6]`
`NullSpace[mat /. T → 10^12]`

$Out[\circ]:=$ $\{35, 35\}$

$Out[\circ]:=$ 32

$Out[\circ]:=$ $\Big\{\Big\{0, 0, 0, \dfrac{333\,333\,333\,334\,000\,000\,000\,001}{222\,222\,222\,222\,222\,222\,222\,222}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,$

$-\dfrac{2\,000\,000\,000\,006\,000\,000\,000\,006\,000\,000\,000\,002}{333\,333\,333\,332\,333\,333\,333\,334\,333\,333\,333\,333}, \dfrac{2\,000\,000\,000\,006\,000\,000\,000\,006\,000\,000\,000\,002}{333\,333\,333\,332\,333\,333\,333\,334\,333\,333\,333\,333},$

$0, 0, \dfrac{272\,727\,272\,728\,181\,818\,181\,819\,818\,181\,818\,182\,727\,272\,727\,273}{30\,303\,030\,303\,030\,303\,030\,302\,969\,696\,969\,696\,969\,696\,969\,697},$

$0, 0, 0, -\dfrac{666\,666\,666\,668\,000\,000\,000\,002}{111\,111\,111\,111\,111\,111\,111\,111}, 0, 0, 0, 0, 1, 0\Big\},$

$\Big\{0, 0, 0, \dfrac{1}{2}, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1,$

$0, 0, 0, -2, 0, 0, 1, 0, 0, 0\Big\}, \Big\{0, 0, \dfrac{1}{2}, 0, 0, 0, 0, 0, 0, 3, 0, -1,$

$0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, -2, 0, 0, 1, 0, 0, 0, 0\Big\}\Big\}$

```
In[ ]:= MatrixForm[mat = Table[
          Factor@{
            OP2[K], c_{2,1}[K], c_{2,2}[K], -2 c_{2,0}[K] + ω[K] c_{2,1}[K],
            ω[K] ρ1[K], ρ1[K], ρ1[K] T ∂_T ω[K], ω[K] T ∂_T ρ1[K],
            ω[K]² T ∂_T ω[K], ω[K] T ∂_T ω[K], T ∂_T ω[K],
            ω[K] (T ∂_T ω[K])², (T ∂_T ω[K])²,
            ω[K]² T² ∂_{T,T} ω[K], ω[K] T² ∂_{T,T} ω[K],
            1
          },
          {K, AllKnots[{8, 8}]}
        ]];
      Dimensions[mat]
      MatrixRank[mat /. T → 10⁴]
      NullSpace[mat /. T → 10⁴] // MatrixForm
```

Out[ ]= {21, 16}

Out[ ]= 14

Out[ ]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -2 & 0 & 1 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 3 & -1 & 1 & 0 & 0 & -2 & 0 & 1 & 0 & 0 \end{pmatrix}$$

```
In[ ]:= Length[AllKnots[{8, 8}]]
```

Out[ ]= 21

```
In[ ]:= MatrixForm[mat = Table[
          Factor@{
            OP2[K], c_{2,1}[K], c_{2,2}[K], -2 c_{2,0}[K] + ω[K] c_{2,1}[K],
            ω[K]² ρ1[K], ω[K] ρ1[K], ρ1[K], ω[K] ρ1[K] T ∂_T ω[K], ρ1[K] T ∂_T ω[K], T ∂_T ρ1[K],
            ω[K] T ∂_T ρ1[K], ω[K]² T ∂_T ρ1[K], T² ∂_{T,T} ρ1[K], ω[K] T² ∂_{T,T} ρ1[K], T² ∂_{T,T} (ω[K] ρ1[K]),
            ρ1[K]², ρ1[K] T ∂_T ρ1[K],
            ω[K]⁴, ω[K]³, ω[K]², ω[K], 1,
            ω[K]³ T ∂_T ω[K], ω[K]² T ∂_T ω[K], ω[K] T ∂_T ω[K], T ∂_T ω[K],
            ω[K]² (T ∂_T ω[K])², ω[K] (T ∂_T ω[K])², (T ∂_T ω[K])²,
            ω[K]³ T² ∂_{T,T} ω[K], ω[K]² T² ∂_{T,T} ω[K], ω[K] T² ∂_{T,T} ω[K], T² ∂_{T,T} ω[K]
          },
          {K, AllKnots[{3, 8}]}
        ]];
      Dimensions[mat]
      MatrixRank[mat /. T → 10⁴]
      NullSpace[mat /. T → 10⁴] // MatrixForm
```

Out[ ]= {35, 33}

Out[ ]= 31

Out[ ]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -2 & 0 & 0 & 1 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 3 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -2 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

```
In[◦]:= MatrixForm@H[mat = Table[
          Factor@{
            -2 c_{2,0}[K] + ω[K] c_{2,1}[K],
            ω[K]^2 ρ1[K], ω[K] ρ1[K], ρ1[K], (T - T^{-1}) ω[K] ρ1[K] T ∂_T ω[K],
            (T - T^{-1}) ρ1[K] T ∂_T ω[K], (T - T^{-1}) T ∂_T ρ1[K], (T - T^{-1}) ω[K] T ∂_T ρ1[K], (T - T^{-1})
             ω[K]^2 T ∂_T ρ1[K], T ∂_T (T ∂_T ρ1[K]), ω[K] T ∂_T (T ∂_T ρ1[K]), T ∂_T (T ∂_T (ω[K] ρ1[K])),
            ρ1[K]^2, ρ1[K] (T - T^{-1}) T ∂_T ρ1[K],
            ω[K]^4, ω[K]^3, ω[K]^2, ω[K],
            (T - T^{-1}) ω[K]^3 T ∂_T ω[K],
            (T - T^{-1}) ω[K]^2 T ∂_T ω[K], (T - T^{-1}) ω[K] T ∂_T ω[K], (T - T^{-1}) T ∂_T ω[K],
            ω[K]^2 (T ∂_T ω[K])^2, ω[K] (T ∂_T ω[K])^2, (T ∂_T ω[K])^2,
            ω[K]^3 T ∂_T (T ∂_T ω[K]), ω[K]^2 T ∂_T (T ∂_T ω[K]), ω[K] T ∂_T (T ∂_T ω[K]), T ∂_T (T ∂_T ω[K]),
            (T - T^{-1}) ω[K] T ∂_T (T ∂_T (T ∂_T ω[K])), (T - T^{-1}) T ∂_T (T ∂_T (T ∂_T ω[K]))
          },
          {K, AllKnots[{3, 9}]}
        ]];
      Dimensions[mat]
      MatrixRank[mat /. T → 10^4]
      NullSpace[mat /. T → 10^4] // MatrixForm

Out[◦]= {84, 31}

Out[◦]= 31

Out[◦]//MatrixForm=
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In[◦]:= (mat0 = Map[Coefficient[#, T, 0] & , mat, {2}]) // MatrixForm
Out[◦]//MatrixForm=
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−510	128	28	6	50	10	12	44	144	0	52	0
−1362	682	−110	18	268	−52	28	−124	604	0	−56	0
−1598	578	74	10	228	36	12	68	428	0	24	0
−6410	1696	−170	16	612	−58	32	−256	1936	0	−200	0
−8510	0	0	0	0	0	0	0	0	0	0	0
−2772	496	−78	12	196	−28	24	−124	576	0	−292	0
−11 718	5472	−564	60	2172	−276	88	−608	4672	0	−264	0
−25 736	−7498	−644	−56	−3110	−298	−104	−908	−7924	0	−968	0
−39 188	−18 800	1424	−112	−7488	704	−160	1504	−15 776	0	640	0
−76 414	25 034	1666	114	10 832	896	204	2140	24 796	0	1936	0
−55 946	21 448	−1278	78	8818	−644	132	−1572	20 624	0	−948	0
−53 622	−5984	−344	−22	−3194	−246	−28	−284	−3600	0	−164	0
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−63 556	12 560	812	50	4686	252	100	1320	14 960	0	2148	0
−20 364	0	0	0	0	0	0	0	0	0	0	0
−53 540	7854	−440	20	2094	−30	48	−812	10 748	0	−640	0
−135 628	−32 378	−1722	−92	−13 148	−808	−176	−2472	−35 844	0	−3476	0
−146 916	49 632	−2428	120	20 436	−1188	216	−3192	50 328	0	−2464	0
−143 216	−23 136	1104	−50	−8294	366	−100	1752	−27 904	0	2276	0
−133 180	−23 952	−1116	−56	−11 628	−724	−88	−1192	−19 544	0	−864	0
−162 396	0	0	0	0	0	0	0	0	0	0	0
−251 346	−47 852	1978	−82	−18 978	912	−156	2820	−53 112	0	3228	0
−263 172	93 594	−3906	166	38 288	−1932	292	−5028	93 796	0	−3516	0
−185 846	0	0	0	0	0	0	0	0	0	0	0
−227 432	−38 986	−1568	−68	−18 690	−998	−104	−1652	−31 532	0	−1096	0

−465 478	179 570	−6582	250	76 064	−3488	420	−8012	170 644	0	−5248	0
−1 228 222	418 246	14 438	520	183 632	8072	888	17 620	396 052	0	13 560	0
−698 666	138 428	−4422	142	55 484	−2068	268	−6236	151 572	0	−6824	0
−717 954	0	0	0	0	0	0	0	0	0	0	0
−1 567 262	0	0	0	0	0	0	0	0	0	0	0
−1060	−110	−24	−6	−42	−24	−4	12	4	0	−112	0
−3640	768	104	16	480	96	24	80	408	0	80	0
−31 226	11 886	−906	72	5280	−512	120	−1068	10 804	0	−744	0
−9780	1360	166	20	540	60	40	268	1600	0	1028	0
−56 420	23 500	−1780	140	9360	−880	200	−1880	19 720	0	−800	0
−160 976	−32 410	1894	−112	−13 446	822	−216	2856	−36 124	0	5928	0
−213 338	61 584	3290	180	26 144	1630	328	4488	63 408	0	4640	0
−300 294	−129 090	6390	−330	−51 540	3180	−460	6620	−106 540	0	2760	0
−659 788	149 930	−6240	264	64 926	−3178	504	−8788	159 660	0	−15 440	0
−819 394	260 376	10 198	414	114 058	5640	724	12 732	251 248	0	10 684	0
−342 134	86 986	−3142	114	33 888	−1440	204	−4148	89 948	0	−2656	0
−1 151 564	281 412	−10 284	388	125 792	−5792	728	−13 736	288 408	0	−21 288	0
−1 259 748	−389 300	−13 316	−468	−167 520	−7008	−840	−17 384	−389 768	0	−16 256	0
−895 336	−227 608	−7702	−260	−93 112	−3462	−496	−11 156	−248 912	0	−14 612	0
−841 572	335 090	−10 900	368	142 102	−5790	608	−13 036	312 988	0	−8048	0
−2 095 210	−663 392	−20 332	−644	−289 684	−11 124	−1136	−25 688	−646 168	0	−22 320	0
−642 168	−174 898	−5542	−190	−83 992	−3508	−276	−5580	−134 772	0	−3316	0
−1 313 504	−543 664	15 932	−488	−233 100	8636	−784	18 512	−494 032	0	10 928	0
−2 949 428	−747 004	21 756	−658	−337 124	12 504	−1220	28 544	−752 376	0	40 376	0
−1 089 796	155 056	−4274	104	46 328	−772	224	−7476	205 600	0	−8164	0
−3 159 722	1 031 298	28 610	826	452 980	15 956	1428	35 324	985 940	0	28 648	0
−836 608	111 696	3084	88	44 692	1516	144	3760	107 856	0	2032	0
−2 080 192	614 504	16 966	482	266 698	9166	884	22 396	626 352	0	24 728	0
−1 868 380	−787 882	20 938	−582	−334 784	11 228	−924	24 188	−714 220	0	13 684	0
−1 540 398	−249 564	6340	−152	−84 974	2034	−308	10 036	−306 120	0	10 372	0
−4 797 258	1 581 696	40 086	1062	698 170	22 610	1804	48 552	1 485 872	0	37 092	0
−1 595 654	84 760	2304	72	53 552	2144	112	1880	47 496	0	2152	0
−2 841 166	1 111 042	−26 960	682	478 872	−14 688	1128	−32 124	1 032 012	0	−20 856	0
−2 381 116	−603 000	14 482	−354	−255 182	7436	−652	19 380	−622 208	0	19 344	0
−2 171 344	164 248	4046	112	96 388	3362	176	3708	106 416	0	3756	0
−3 186 008	862 254	−19 284	450	382 198	−10 880	796	−24 340	842 236	0	−22 840	0
−2 840 192	499 442	−10 804	230	179 982	−4284	444	−15 988	583 220	0	−14 512	0
−2 863 228	206 978	4644	114	112 408	3438	184	4712	152 316	0	4340	0
−4 319 004	1 256 080	−26 110	568	558 272	−14 808	992	−32 524	1 211 744	0	−28 668	0
−5 736 454	−1 485 840	28 510	−562	−635 112	15 052	−1016	37 324	−1 505 496	0	35 032	0
−5 100 726	323 596	6376	140	185 992	5144	220	5952	215 212	0	5744	0
−8 043 256	625 898	10 706	198	325 562	7450	320	11 248	483 424	0	9488	0
−596 734	251 712	−10 620	468	100 548	−5292	648	−10 944	206 784	0	−4536	0
−1 372 104	−374 394	−11 368	−350	−156 988	−5662	−656	−15 712	−397 500	0	−18 828	0
−1 204 192	151 274	3816	100	60 938	1902	160	4556	143 380	0	2384	0
−12 103 772	4 000 568	79 972	1668	1 755 976	44 688	2840	97 312	3 777 224	0	73 840	0
−5 255 776	−2 120 644	44 032	−956	−907 112	23 816	−1544	51 576	−1 944 944	0	30 976	0
−14 730 526	4 112 120	−62 430	986	1 796 314	−34 472	1732	−78 740	4 023 528	0	−68 180	0
−2 129 794	582 566	13 874	356	277 872	8736	536	14 468	465 908	0	9144	0
−3128	92	−18	10	156	−88	8	56	−176	0	16	0
−28 462	−3144	−226	−18	−828	142	−32	−632	−4320	0	−880	0
−32 126	8394	576	42	3976	352	64	604	6652	0	360	0
−200 284	82 084	−4094	214	35 740	−2264	336	−4608	71 952	0	−2624	0
−4222	1734	222	30	684	108	36	204	1284	0	72	0
−288 924	−34 662	1356	−44	−8890	−46	−96	2708	−49 528	0	3336	0
−241 202	−101 184	4274	−188	−42 184	2230	−296	4920	−91 664	0	2608	0
−373 944	−117 224	−5282	−242	−49 596	−2688	−440	−6992	−119 096	0	−6432	0

`In[ ]:=` **s = 10^5;**
(red = LatticeReduce[Prepend[mat0, s IdentityMatrix[Dimensions[mat0][[2]]][[1]]]$^\top$]$^\top$);
rel = LinearSolve[mat0, Rest[red][[All, Position[red[[1]], s][[1, 1]]]]]
Factor@Expand[mat.rel] // Column

`Out[ ]=` {1, -1, 94, -1938, 2, -40, 1181, -88, 0, 0, 31, 0, 13, -13, 9, 312,
-982, -11 259, -22, -539, -188, 8370, -9, -18, -1715, 0, 0, 0, 0, -255, 0}

$$-\frac{46+605\,T-3718\,T^2+99\,T^3+6374\,T^4+4988\,T^5+6374\,T^6+99\,T^7-3718\,T^8+605\,T^9+46\,T^{10}}{T^5}$$

$$-\frac{22-738\,T+4644\,T^2+1113\,T^3-10\,812\,T^4+23\,458\,T^5-10\,812\,T^6+1113\,T^7+4644\,T^8-738\,T^9+22\,T^{10}}{T^5}$$

$$-\frac{1}{T^9}\left(244 - 877\,T + 3499\,T^2 - 8295\,T^3 + 4197\,T^4 + 11\,268\,T^5 - 24\,794\,T^6 + 20\,511\,T^7 + 7452\,T^8 - 15\,098\,T^9 + \right.$$
$$\left. 7452\,T^{10} + 20\,511\,T^{11} - 24\,794\,T^{12} + 11\,268\,T^{13} + 4197\,T^{14} - 8295\,T^{15} + 3499\,T^{16} - 877\,T^{17} + 244\,T^{18}\right)$$

$$-\frac{962+2035\,T-25\,020\,T^2+40\,618\,T^3-18\,434\,T^4+11\,234\,T^5-18\,434\,T^6+40\,618\,T^7-25\,020\,T^8+2035\,T^9+962\,T^{10}}{T^5}$$

$$-\frac{370-7573\,T+34\,484\,T^2-30\,340\,T^3-10\,290\,T^4+38\,734\,T^5-10\,290\,T^6-30\,340\,T^7+34\,484\,T^8-7573\,T^9+370\,T^{10}}{T^5}$$

$$-\frac{1}{T^9}\left(92 - 954\,T + 2753\,T^2 + 1127\,T^3 - 24\,011\,T^4 + 74\,708\,T^5 - 105\,408\,T^6 + 65\,811\,T^7 - 13\,460\,T^8 + 10\,724\,T^9 - \right.$$
$$\left. 13\,460\,T^{10} + 65\,811\,T^{11} - 105\,408\,T^{12} + 74\,708\,T^{13} - 24\,011\,T^{14} + 1127\,T^{15} + 2753\,T^{16} - 954\,T^{17} + 92\,T^{18}\right)$$

$$-\frac{1}{T^9}\left(44 - 439\,T + 3293\,T^2 - 15\,487\,T^3 + 50\,059\,T^4 - 98\,589\,T^5 + 103\,566\,T^6 - 44\,635\,T^7 - 4884\,T^8 + 26\,068\,T^9 - \right.$$
$$\left. 4884\,T^{10} - 44\,635\,T^{11} + 103\,566\,T^{12} - 98\,589\,T^{13} + 50\,059\,T^{14} - 15\,487\,T^{15} + 3293\,T^{16} - 439\,T^{17} + 44\,T^{18}\right)$$

$$-\frac{1}{T^{13}}\left(750 - 2792\,T + 5331\,T^2 - 4133\,T^3 - 3755\,T^4 + 11\,444\,T^5 - 30\,997\,T^6 + 68\,833\,T^7 - 70\,664\,T^8 + 33\,188\,T^9 - \right.$$
$$11\,047\,T^{10} + 17\,089\,T^{11} + 2558\,T^{12} - 21\,402\,T^{13} + 2558\,T^{14} + 17\,089\,T^{15} - 11\,047\,T^{16} + 33\,188\,T^{17} - $$
$$\left. 70\,664\,T^{18} + 68\,833\,T^{19} - 30\,997\,T^{20} + 11\,444\,T^{21} - 3755\,T^{22} - 4133\,T^{23} + 5331\,T^{24} - 2792\,T^{25} + 750\,T^{26}\right)$$

$$-\frac{6626-10\,390\,T-51\,052\,T^2+129\,549\,T^3-85\,272\,T^4+32\,266\,T^5-85\,272\,T^6+129\,549\,T^7-51\,052\,T^8-10\,390\,T^9+6626\,T^{10}}{T^5}$$

$$-\frac{1}{T^9}\left(5060 - 26\,369\,T + 67\,560\,T^2 - 108\,276\,T^3 + 182\,226\,T^4 - 223\,237\,T^5 + \right.$$
$$125\,996\,T^6 - 15\,253\,T^7 + 8220\,T^8 - 18\,550\,T^9 + 8220\,T^{10} - 15\,253\,T^{11} + 125\,996\,T^{12} - $$
$$\left. 223\,237\,T^{13} + 182\,226\,T^{14} - 108\,276\,T^{15} + 67\,560\,T^{16} - 26\,369\,T^{17} + 5060\,T^{18}\right)$$

$$-\frac{2\left(10\,688-51\,056\,T+137\,728\,T^2-181\,579\,T^3+84\,640\,T^4+5622\,T^5+84\,640\,T^6-181\,579\,T^7+137\,728\,T^8-51\,056\,T^9+10\,688\,T^{10}\right)}{T^5}$$

$$-\frac{1}{T^9}\left(4772 - 32\,365\,T + 125\,512\,T^2 - 335\,032\,T^3 + 604\,812\,T^4 - 738\,146\,T^5 + \right.$$
$$611\,296\,T^6 - 327\,124\,T^7 + 97\,808\,T^8 - 12\,122\,T^9 + 97\,808\,T^{10} - 327\,124\,T^{11} + 611\,296\,T^{12} - $$
$$\left. 738\,146\,T^{13} + 604\,812\,T^{14} - 335\,032\,T^{15} + 125\,512\,T^{16} - 32\,365\,T^{17} + 4772\,T^{18}\right)$$

$$-\frac{1}{T^9}\left(92 - 1598\,T + 10\,373\,T^2 - 30\,383\,T^3 + 26\,473\,T^4 + 90\,023\,T^5 - \right.$$
$$301\,702\,T^6 + 358\,209\,T^7 - 170\,894\,T^8 + 50\,490\,T^9 - 170\,894\,T^{10} + 358\,209\,T^{11} - $$
$$\left. 301\,702\,T^{12} + 90\,023\,T^{13} + 26\,473\,T^{14} - 30\,383\,T^{15} + 10\,373\,T^{16} - 1598\,T^{17} + 92\,T^{18}\right)$$

$$-\frac{1}{T^9}\left(44 - 747\,T + 6879\,T^2 - 40\,655\,T^3 + 160\,193\,T^4 - 407\,688\,T^5 + \right.$$
$$628\,218\,T^6 - 522\,735\,T^7 + 158\,480\,T^8 + 47\,814\,T^9 + 158\,480\,T^{10} - 522\,735\,T^{11} + $$
$$\left. 628\,218\,T^{12} - 407\,688\,T^{13} + 160\,193\,T^{14} - 40\,655\,T^{15} + 6879\,T^{16} - 747\,T^{17} + 44\,T^{18}\right)$$

$$-\frac{2342-34\,981\,T+136\,294\,T^2-158\,599\,T^3+35\,618\,T^4+50\,932\,T^5+35\,618\,T^6-158\,599\,T^7+136\,294\,T^8-34\,981\,T^9+2342\,T^{10}}{T^5}$$

$$-\frac{1}{T^{13}}\left(366 - 4009\,T + 19\,193\,T^2 - 57\,167\,T^3 + 130\,561\,T^4 - 250\,012\,T^5 + 385\,365\,T^6 - \right.$$
$$433\,749\,T^7 + 354\,612\,T^8 - 247\,990\,T^9 + 178\,053\,T^{10} - 147\,083\,T^{11} + 132\,494\,T^{12} - 109\,224\,T^{13} + $$
$$132\,494\,T^{14} - 147\,083\,T^{15} + 178\,053\,T^{16} - 247\,990\,T^{17} + 354\,612\,T^{18} - 433\,749\,T^{19} + $$
$$\left. 385\,365\,T^{20} - 250\,012\,T^{21} + 130\,561\,T^{22} - 57\,167\,T^{23} + 19\,193\,T^{24} - 4009\,T^{25} + 366\,T^{26}\right)$$

$$-\frac{2\left(2816-36\,368\,T+141\,448\,T^2-186\,961\,T^3+76\,456\,T^4+11\,170\,T^5+76\,456\,T^6-186\,961\,T^7+141\,448\,T^8-36\,368\,T^9+2816\,T^{10}\right)}{T^5}$$

$-\frac{1}{T^9}\Big(1124 - 9305\,T + 23\,164\,T^2 - 1634\,T^3 - 101\,074\,T^4 +$
$277\,963\,T^5 - 366\,100\,T^6 + 238\,191\,T^7 - 55\,972\,T^8 - 682\,T^9 - 55\,972\,T^{10} + 238\,191\,T^{11} -$
$366\,100\,T^{12} + 277\,963\,T^{13} - 101\,074\,T^{14} - 1634\,T^{15} + 23\,164\,T^{16} - 9305\,T^{17} + 1124\,T^{18}\Big)$

$-\frac{1}{T^{13}}\Big(390 - 4277\,T + 21\,547\,T^2 - 68\,447\,T^3 + 156\,036\,T^4 - 273\,626\,T^5 + 411\,409\,T^6 -$
$557\,353\,T^7 + 618\,416\,T^8 - 501\,909\,T^9 + 274\,098\,T^{10} - 96\,304\,T^{11} + 20\,166\,T^{12} + 11\,252\,T^{13} +$
$20\,166\,T^{14} - 96\,304\,T^{15} + 274\,098\,T^{16} - 501\,909\,T^{17} + 618\,416\,T^{18} - 557\,353\,T^{19} +$
$411\,409\,T^{20} - 273\,626\,T^{21} + 156\,036\,T^{22} - 68\,447\,T^{23} + 21\,547\,T^{24} - 4277\,T^{25} + 390\,T^{26}\Big)$

$-\frac{1}{T^9}\Big(1924 - 20\,113\,T + 81\,058\,T^2 - 158\,106\,T^3 + 116\,828\,T^4 + 172\,019\,T^5 -$
$499\,230\,T^6 + 474\,462\,T^7 - 171\,424\,T^8 + 17\,448\,T^9 - 171\,424\,T^{10} + 474\,462\,T^{11} -$
$499\,230\,T^{12} + 172\,019\,T^{13} + 116\,828\,T^{14} - 158\,106\,T^{15} + 81\,058\,T^{16} - 20\,113\,T^{17} + 1924\,T^{18}\Big)$

$-\frac{1}{T^{13}}\Big(150 - 1600\,T + 8641\,T^2 - 29\,507\,T^3 + 71\,017\,T^4 - 127\,861\,T^5 + 193\,313\,T^6 -$
$246\,765\,T^7 + 248\,472\,T^8 - 185\,466\,T^9 + 98\,383\,T^{10} - 41\,279\,T^{11} + 34\,336\,T^{12} - 31\,492\,T^{13} +$
$34\,336\,T^{14} - 41\,279\,T^{15} + 98\,383\,T^{16} - 185\,466\,T^{17} + 248\,472\,T^{18} - 246\,765\,T^{19} +$
$193\,313\,T^{20} - 127\,861\,T^{21} + 71\,017\,T^{22} - 29\,507\,T^{23} + 8641\,T^{24} - 1600\,T^{25} + 150\,T^{26}\Big)$

$-\frac{1}{T^9}\Big(772 - 7753\,T + 46\,210\,T^2 - 180\,562\,T^3 + 493\,916\,T^4 - 903\,157\,T^5 +$
$1\,054\,902\,T^6 - 726\,178\,T^7 + 207\,988\,T^8 + 39\,776\,T^9 + 207\,988\,T^{10} - 726\,178\,T^{11} +$
$1\,054\,902\,T^{12} - 903\,157\,T^{13} + 493\,916\,T^{14} - 180\,562\,T^{15} + 46\,210\,T^{16} - 7753\,T^{17} + 772\,T^{18}\Big)$

$-\frac{1}{T^{13}}\Big(66 - 663\,T + 3389\,T^2 - 13\,539\,T^3 + 45\,233\,T^4 - 122\,616\,T^5 + 277\,433\,T^6 -$
$497\,091\,T^7 + 687\,110\,T^8 - 717\,955\,T^9 + 545\,449\,T^{10} - 289\,723\,T^{11} + 97\,648\,T^{12} - 17\,570\,T^{13} +$
$97\,648\,T^{14} - 289\,723\,T^{15} + 545\,449\,T^{16} - 717\,955\,T^{17} + 687\,110\,T^{18} - 497\,091\,T^{19} +$
$277\,433\,T^{20} - 122\,616\,T^{21} + 45\,233\,T^{22} - 13\,539\,T^{23} + 3389\,T^{24} - 663\,T^{25} + 66\,T^{26}\Big)$

$-\frac{1}{T^{13}}\Big(1 - T + T^2\Big)$
$\Big(150 - 1450\,T + 7405\,T^2 - 24\,246\,T^3 + 57\,285\,T^4 - 101\,924\,T^5 + 153\,746\,T^6 - 180\,397\,T^7 + 153\,269\,T^8 -$
$89\,240\,T^9 + 21\,749\,T^{10} + 2551\,T^{11} + 14\,508\,T^{12} + 2551\,T^{13} + 21\,749\,T^{14} - 89\,240\,T^{15} + 153\,269\,T^{16} -$
$180\,397\,T^{17} + 153\,746\,T^{18} - 101\,924\,T^{19} + 57\,285\,T^{20} - 24\,246\,T^{21} + 7405\,T^{22} - 1450\,T^{23} + 150\,T^{24}\Big)$

$-\frac{1}{T^9}\Big(1924 - 23\,041\,T + 109\,606\,T^2 - 266\,018\,T^3 + 318\,200\,T^4 - 1300\,T^5 -$
$520\,432\,T^6 + 641\,479\,T^7 - 277\,372\,T^8 + 46\,072\,T^9 - 277\,372\,T^{10} + 641\,479\,T^{11} -$
$520\,432\,T^{12} - 1300\,T^{13} + 318\,200\,T^{14} - 266\,018\,T^{15} + 109\,606\,T^{16} - 23\,041\,T^{17} + 1924\,T^{18}\Big)$

$-\frac{1}{T^9}\Big(44 - 1055\,T + 12\,025\,T^2 - 83\,585\,T^3 + 377\,999\,T^4 - 1\,101\,975\,T^5 +$
$1\,947\,258\,T^6 - 1\,860\,359\,T^7 + 708\,128\,T^8 + 14\,948\,T^9 + 708\,128\,T^{10} - 1\,860\,359\,T^{11} +$
$1\,947\,258\,T^{12} - 1\,101\,975\,T^{13} + 377\,999\,T^{14} - 83\,585\,T^{15} + 12\,025\,T^{16} - 1055\,T^{17} + 44\,T^{18}\Big)$

$-\frac{1}{T^9}\Big(772 - 8997\,T + 58\,436\,T^2 - 247\,904\,T^3 + 729\,062\,T^4 - 1\,442\,785\,T^5 +$
$1\,837\,964\,T^6 - 1\,381\,061\,T^7 + 437\,148\,T^8 + 46\,778\,T^9 + 437\,148\,T^{10} - 1\,381\,061\,T^{11} +$
$1\,837\,964\,T^{12} - 1\,442\,785\,T^{13} + 729\,062\,T^{14} - 247\,904\,T^{15} + 58\,436\,T^{16} - 8997\,T^{17} + 772\,T^{18}\Big)$

$-\frac{1}{T^9}\Big(1924 - 25\,969\,T + 143\,488\,T^2 - 422\,704\,T^3 + 697\,708\,T^4 - 515\,540\,T^5 -$
$184\,136\,T^6 + 620\,522\,T^7 - 347\,120\,T^8 + 75\,574\,T^9 - 347\,120\,T^{10} + 620\,522\,T^{11} - 184\,136\,T^{12} -$
$515\,540\,T^{13} + 697\,708\,T^{14} - 422\,704\,T^{15} + 143\,488\,T^{16} - 25\,969\,T^{17} + 1924\,T^{18}\Big)$

$-\frac{1}{T^9}\Big(25\,740 - 232\,224\,T + 1\,070\,486\,T^2 - 3\,230\,996\,T^3 + 6\,809\,348\,T^4 - 10\,166\,915\,T^5 + 10\,585\,546\,T^6 -$
$7\,095\,673\,T^7 + 2\,209\,500\,T^8 + 61\,444\,T^9 + 2\,209\,500\,T^{10} - 7\,095\,673\,T^{11} + 10\,585\,546\,T^{12} -$
$10\,166\,915\,T^{13} + 6\,809\,348\,T^{14} - 3\,230\,996\,T^{15} + 1\,070\,486\,T^{16} - 232\,224\,T^{17} + 25\,740\,T^{18}\Big)$

$-\frac{1}{T^{13}}\Big(138 - 1972\,T + 13\,690\,T^2 - 58\,511\,T^3 + 167\,265\,T^4 - 326\,571\,T^5 + 416\,837\,T^6 -$
$233\,089\,T^7 - 330\,749\,T^8 + 1\,012\,873\,T^9 - 1\,270\,987\,T^{10} + 874\,235\,T^{11} - 214\,604\,T^{12} - 85\,310\,T^{13} -$
$214\,604\,T^{14} + 874\,235\,T^{15} - 1\,270\,987\,T^{16} + 1\,012\,873\,T^{17} - 330\,749\,T^{18} - 233\,089\,T^{19} +$
$416\,837\,T^{20} - 326\,571\,T^{21} + 167\,265\,T^{22} - 58\,511\,T^{23} + 13\,690\,T^{24} - 1972\,T^{25} + 138\,T^{26}\Big)$

$$-\frac{1}{T^{13}}\Big(66 - 905\,T + 6154\,T^2 - 29\,325\,T^3 + 109\,735\,T^4 - 331\,701\,T^5 + 821\,637\,T^6 - 1\,641\,859\,T^7 +$$
$$2\,587\,135\,T^8 - 3\,129\,293\,T^9 + 2\,782\,351\,T^{10} - 1\,663\,099\,T^{11} + 495\,280\,T^{12} - 436\,T^{13} +$$
$$495\,280\,T^{14} - 1\,663\,099\,T^{15} + 2\,782\,351\,T^{16} - 3\,129\,293\,T^{17} + 2\,587\,135\,T^{18} - 1\,641\,859\,T^{19} +$$
$$821\,637\,T^{20} - 331\,701\,T^{21} + 109\,735\,T^{22} - 29\,325\,T^{23} + 6154\,T^{24} - 905\,T^{25} + 66\,T^{26}\Big)$$

$$-\frac{1}{T^{13}}\Big(1 - T + T^2\Big)\Big(66 - 1081\,T + 8212\,T^2 - 40\,438\,T^3 + 149\,094\,T^4 - 432\,711\,T^5 +$$
$$997\,160\,T^6 - 1\,785\,744\,T^7 + 2\,366\,156\,T^8 - 2\,161\,094\,T^9 + 1\,172\,396\,T^{10} - 150\,426\,T^{11} -$$
$$231\,256\,T^{12} - 150\,426\,T^{13} + 1\,172\,396\,T^{14} - 2\,161\,094\,T^{15} + 2\,366\,156\,T^{16} - 1\,785\,744\,T^{17} +$$
$$997\,160\,T^{18} - 432\,711\,T^{19} + 149\,094\,T^{20} - 40\,438\,T^{21} + 8212\,T^{22} - 1081\,T^{23} + 66\,T^{24}\Big)$$

$$-\frac{1}{T^{13}}\Big(786 - 2930\,T + 3383\,T^2 + 1419\,T^3 - 3622\,T^4 - 3700\,T^5 + 40\,383\,T^6 - 40\,819\,T^7 - 29\,416\,T^8 + 52\,113\,T^9 -$$
$$6500\,T^{10} - 15\,826\,T^{11} + 23\,776\,T^{12} - 24\,914\,T^{13} + 23\,776\,T^{14} - 15\,826\,T^{15} - 6500\,T^{16} + 52\,113\,T^{17} -$$
$$29\,416\,T^{18} - 40\,819\,T^{19} + 40\,383\,T^{20} - 3700\,T^{21} - 3622\,T^{22} + 1419\,T^{23} + 3383\,T^{24} - 2930\,T^{25} + 786\,T^{26}\Big)$$

$$-\frac{1}{T^9}\Big(44 - 285\,T + 2084\,T^2 - 8008\,T^3 + 23\,582\,T^4 - 37\,474\,T^5 + 13\,126\,T^6 + 25\,441\,T^7 - 15\,540\,T^8 + 5740\,T^9 -$$
$$15\,540\,T^{10} + 25\,441\,T^{11} + 13\,126\,T^{12} - 37\,474\,T^{13} + 23\,582\,T^{14} - 8008\,T^{15} + 2084\,T^{16} - 285\,T^{17} + 44\,T^{18}\Big)$$

$$-\frac{1}{T^9}$$
$$\Big(92 - 1450\,T + 8258\,T^2 - 20\,630\,T^3 + 13\,404\,T^4 + 68\,079\,T^5 - 193\,706\,T^6 + 206\,355\,T^7 - 87\,572\,T^8 + 26\,384\,T^9 -$$
$$87\,572\,T^{10} + 206\,355\,T^{11} - 193\,706\,T^{12} + 68\,079\,T^{13} + 13\,404\,T^{14} - 20\,630\,T^{15} + 8258\,T^{16} - 1450\,T^{17} + 92\,T^{18}\Big)$$

$$-\frac{1}{T^{17}}\Big(1720 - 6519\,T + 12\,949\,T^2 - 19\,728\,T^3 + 31\,619\,T^4 - 54\,841\,T^5 + 77\,764\,T^6 -$$
$$96\,297\,T^7 + 88\,716\,T^8 - 38\,073\,T^9 + 27\,299\,T^{10} - 30\,455\,T^{11} + 6810\,T^{12} - 18\,028\,T^{13} +$$
$$64\,181\,T^{14} - 70\,065\,T^{15} + 74\,366\,T^{16} - 94\,596\,T^{17} + 74\,366\,T^{18} - 70\,065\,T^{19} + 64\,181\,T^{20} -$$
$$18\,028\,T^{21} + 6810\,T^{22} - 30\,455\,T^{23} + 27\,299\,T^{24} - 38\,073\,T^{25} + 88\,716\,T^{26} - 96\,297\,T^{27} +$$
$$77\,764\,T^{28} - 54\,841\,T^{29} + 31\,619\,T^{30} - 19\,728\,T^{31} + 12\,949\,T^{32} - 6519\,T^{33} + 1720\,T^{34}\Big)$$

$$-\frac{4\,\Big(7018 - 22\,941\,T + 5777\,T^2 + 42\,741\,T^3 - 37\,549\,T^4 + 12\,582\,T^5 - 37\,549\,T^6 + 42\,741\,T^7 + 5777\,T^8 - 22\,941\,T^9 + 7018\,T^{10}\Big)}{T^5}$$

$$-\frac{1}{T^{13}}\Big(14\,550 - 79\,213\,T + 213\,350\,T^2 - 398\,052\,T^3 + 622\,616\,T^4 - 855\,661\,T^5 + 1\,153\,927\,T^6 -$$
$$1\,340\,853\,T^7 + 1\,213\,075\,T^8 - 970\,168\,T^9 + 707\,430\,T^{10} - 389\,443\,T^{11} + 179\,674\,T^{12} - 127\,284\,T^{13} +$$
$$179\,674\,T^{14} - 389\,443\,T^{15} + 707\,430\,T^{16} - 970\,168\,T^{17} + 1\,213\,075\,T^{18} - 1\,340\,853\,T^{19} +$$
$$1\,153\,927\,T^{20} - 855\,661\,T^{21} + 622\,616\,T^{22} - 398\,052\,T^{23} + 213\,350\,T^{24} - 79\,213\,T^{25} + 14\,550\,T^{26}\Big)$$

$$-\frac{1}{T^9}\Big(30\,244 - 175\,202\,T + 546\,341\,T^2 - 1\,182\,525\,T^3 + 1\,771\,117\,T^4 - 1\,853\,700\,T^5 +$$
$$1\,457\,940\,T^6 - 839\,869\,T^7 + 273\,892\,T^8 - 46\,884\,T^9 + 273\,892\,T^{10} - 839\,869\,T^{11} + 1\,457\,940\,T^{12} -$$
$$1\,853\,700\,T^{13} + 1\,771\,117\,T^{14} - 1\,182\,525\,T^{15} + 546\,341\,T^{16} - 175\,202\,T^{17} + 30\,244\,T^{18}\Big)$$

$$-\frac{143\,042 - 786\,853\,T + 1\,986\,120\,T^2 - 2\,453\,846\,T^3 + 1\,213\,314\,T^4 - 189\,750\,T^5 + 1\,213\,314\,T^6 - 2\,453\,846\,T^7 + 1\,986\,120\,T^8 - 786\,853\,T^9 + 143\,042\,T^{10}}{T^5}$$

$$-\frac{1}{T^{13}}\Big(13\,926 - 99\,421\,T + 357\,846\,T^2 - 842\,496\,T^3 + 1\,439\,002\,T^4 - 1\,922\,378\,T^5 + 2\,060\,006\,T^6 -$$
$$1\,686\,340\,T^7 + 980\,750\,T^8 - 354\,097\,T^9 - 2948\,T^{10} + 152\,855\,T^{11} - 149\,368\,T^{12} + 115\,042\,T^{13} -$$
$$149\,368\,T^{14} + 152\,855\,T^{15} - 2948\,T^{16} - 354\,097\,T^{17} + 980\,750\,T^{18} - 1\,686\,340\,T^{19} + 2\,060\,006\,T^{20} -$$
$$1\,922\,378\,T^{21} + 1\,439\,002\,T^{22} - 842\,496\,T^{23} + 357\,846\,T^{24} - 99\,421\,T^{25} + 13\,926\,T^{26}\Big)$$

$$-\frac{1}{T^9}\Big(30\,244 - 240\,286\,T + 990\,389\,T^2 - 2\,709\,811\,T^3 + 5\,202\,601\,T^4 - 7\,116\,345\,T^5 + 6\,880\,562\,T^6 -$$
$$4\,363\,815\,T^7 + 1\,304\,770\,T^8 + 53\,834\,T^9 + 1\,304\,770\,T^{10} - 4\,363\,815\,T^{11} + 6\,880\,562\,T^{12} -$$
$$7\,116\,345\,T^{13} + 5\,202\,601\,T^{14} - 2\,709\,811\,T^{15} + 990\,389\,T^{16} - 240\,286\,T^{17} + 30\,244\,T^{18}\Big)$$

$$-\frac{1}{T^9}\Big(1124 - 14\,949\,T + 77\,508\,T^2 - 195\,460\,T^3 + 205\,576\,T^4 + 167\,638\,T^5 -$$
$$782\,680\,T^6 + 938\,316\,T^7 - 458\,184\,T^8 + 133\,894\,T^9 - 458\,184\,T^{10} + 938\,316\,T^{11} -$$
$$782\,680\,T^{12} + 167\,638\,T^{13} + 205\,576\,T^{14} - 195\,460\,T^{15} + 77\,508\,T^{16} - 14\,949\,T^{17} + 1124\,T^{18}\Big)$$

Out[•]=

$$-\frac{1}{T^{13}}\Big(13\,926 - 99\,421\,T + 376\,058\,T^2 - 973\,088\,T^3 + 1\,903\,688\,T^4 - 2\,994\,990\,T^5 + 3\,842\,358\,T^6 - 3\,972\,538\,T^7 +$$
$$3\,323\,852\,T^8 - 2\,234\,489\,T^9 + 1\,126\,304\,T^{10} - 296\,232\,T^{11} - 130\,866\,T^{12} + 240\,100\,T^{13} -$$
$$130\,866\,T^{14} - 296\,232\,T^{15} + 1\,126\,304\,T^{16} - 2\,234\,489\,T^{17} + 3\,323\,852\,T^{18} - 3\,972\,538\,T^{19} +$$
$$3\,842\,358\,T^{20} - 2\,994\,990\,T^{21} + 1\,903\,688\,T^{22} - 973\,088\,T^{23} + 376\,058\,T^{24} - 99\,421\,T^{25} + 13\,926\,T^{26}\Big)$$

$$-\frac{1}{T^9}2\Big(48\,512 - 331\,728\,T + 1\,095\,536\,T^2 - 2\,312\,368\,T^3 + 3\,636\,636\,T^4 - 4\,365\,995\,T^5 + 3\,813\,258\,T^6 -$$
$$2\,214\,181\,T^7 + 622\,758\,T^8 + 22\,484\,T^9 + 622\,758\,T^{10} - 2\,214\,181\,T^{11} + 3\,813\,258\,T^{12} -$$
$$4\,365\,995\,T^{13} + 3\,636\,636\,T^{14} - 2\,312\,368\,T^{15} + 1\,095\,536\,T^{16} - 331\,728\,T^{17} + 48\,512\,T^{18}\Big)$$

$$-\frac{1}{T^{13}}\Big(390 - 7137\,T + 56\,637\,T^2 - 260\,501\,T^3 + 787\,671\,T^4 - 1\,693\,063\,T^5 + 2\,768\,599\,T^6 - 3\,703\,553\,T^7 +$$
$$4\,184\,956\,T^8 - 3\,790\,922\,T^9 + 2\,547\,677\,T^{10} - 1\,171\,033\,T^{11} + 254\,546\,T^{12} + 64\,518\,T^{13} +$$
$$254\,546\,T^{14} - 1\,171\,033\,T^{15} + 2\,547\,677\,T^{16} - 3\,790\,922\,T^{17} + 4\,184\,956\,T^{18} - 3\,703\,553\,T^{19} +$$
$$2\,768\,599\,T^{20} - 1\,693\,063\,T^{21} + 787\,671\,T^{22} - 260\,501\,T^{23} + 56\,637\,T^{24} - 7137\,T^{25} + 390\,T^{26}\Big)$$

$$-\frac{1}{T^9}\Big(1924 - 30\,653\,T + 200\,196\,T^2 - 703\,584\,T^3 + 1\,446\,782\,T^4 - 1\,678\,981\,T^5 +$$
$$850\,032\,T^6 + 148\,915\,T^7 - 271\,376\,T^8 + 85\,042\,T^9 - 271\,376\,T^{10} + 148\,915\,T^{11} + 850\,032\,T^{12} -$$
$$1\,678\,981\,T^{13} + 1\,446\,782\,T^{14} - 703\,584\,T^{15} + 200\,196\,T^{16} - 30\,653\,T^{17} + 1924\,T^{18}\Big)$$

$$-\frac{1}{T^9}\Big(97\,024 - 748\,352\,T + 2\,795\,744\,T^2 - 6\,645\,694\,T^3 + 11\,448\,746\,T^4 - 14\,732\,638\,T^5 + 13\,761\,972\,T^6 -$$
$$8\,560\,127\,T^7 + 2\,607\,136\,T^8 - 33\,442\,T^9 + 2\,607\,136\,T^{10} - 8\,560\,127\,T^{11} + 13\,761\,972\,T^{12} -$$
$$14\,732\,638\,T^{13} + 11\,448\,746\,T^{14} - 6\,645\,694\,T^{15} + 2\,795\,744\,T^{16} - 748\,352\,T^{17} + 97\,024\,T^{18}\Big)$$

$$-\frac{1}{T^9}\Big(772 - 11\,913\,T + 93\,034\,T^2 - 457\,590\,T^3 + 1\,513\,556\,T^4 - 3\,359\,952\,T^5 +$$
$$4\,824\,040\,T^6 - 4\,092\,597\,T^7 + 1\,532\,028\,T^8 - 71\,088\,T^9 + 1\,532\,028\,T^{10} - 4\,092\,597\,T^{11} +$$
$$4\,824\,040\,T^{12} - 3\,359\,952\,T^{13} + 1\,513\,556\,T^{14} - 457\,590\,T^{15} + 93\,034\,T^{16} - 11\,913\,T^{17} + 772\,T^{18}\Big)$$

$$-\frac{1}{T^9}\Big(2084 - 36\,421\,T + 273\,610\,T^2 - 1\,174\,142\,T^3 + 3\,247\,756\,T^4 - 6\,097\,723\,T^5 + 7\,697\,206\,T^6 -$$
$$5\,971\,346\,T^7 + 1\,997\,644\,T^8 + 135\,212\,T^9 + 1\,997\,644\,T^{10} - 5\,971\,346\,T^{11} + 7\,697\,206\,T^{12} -$$
$$6\,097\,723\,T^{13} + 3\,247\,756\,T^{14} - 1\,174\,142\,T^{15} + 273\,610\,T^{16} - 36\,421\,T^{17} + 2084\,T^{18}\Big)$$

$$-\frac{1}{T^{13}}$$
$$\Big(14\,550 - 128\,797\,T + 586\,012\,T^2 - 1\,820\,904\,T^3 + 4\,342\,938\,T^4 - 8\,398\,092\,T^5 + 13\,658\,066\,T^6 - 18\,829\,334\,T^7 +$$
$$21\,847\,110\,T^8 - 20\,998\,107\,T^9 + 16\,044\,376\,T^{10} - 8\,738\,333\,T^{11} + 2\,298\,396\,T^{12} + 257\,918\,T^{13} +$$
$$2\,298\,396\,T^{14} - 8\,738\,333\,T^{15} + 16\,044\,376\,T^{16} - 20\,998\,107\,T^{17} + 21\,847\,110\,T^{18} - 18\,829\,334\,T^{19} +$$
$$13\,658\,066\,T^{20} - 8\,398\,092\,T^{21} + 4\,342\,938\,T^{22} - 1\,820\,904\,T^{23} + 586\,012\,T^{24} - 128\,797\,T^{25} + 14\,550\,T^{26}\Big)$$

$$-\frac{1}{T^{13}}\Big(138 - 2478\,T + 20\,075\,T^2 - 95\,205\,T^3 + 287\,005\,T^4 - 555\,316\,T^5 + 636\,203\,T^6 - 202\,699\,T^7 -$$
$$745\,048\,T^8 + 1\,661\,292\,T^9 - 1\,775\,755\,T^{10} + 1\,019\,411\,T^{11} - 136\,290\,T^{12} - 210\,754\,T^{13} -$$
$$136\,290\,T^{14} + 1\,019\,411\,T^{15} - 1\,775\,755\,T^{16} + 1\,661\,292\,T^{17} - 745\,048\,T^{18} - 202\,699\,T^{19} +$$
$$636\,203\,T^{20} - 555\,316\,T^{21} + 287\,005\,T^{22} - 95\,205\,T^{23} + 20\,075\,T^{24} - 2478\,T^{25} + 138\,T^{26}\Big)$$

$$-\frac{1}{T^9}\Big(91\,904 - 773\,888\,T + 3\,296\,960\,T^2 - 9\,205\,508\,T^3 + 18\,116\,522\,T^4 - 25\,640\,143\,T^5 + 25\,770\,076\,T^6 -$$
$$16\,961\,364\,T^7 + 5\,239\,326\,T^8 + 142\,314\,T^9 + 5\,239\,326\,T^{10} - 16\,961\,364\,T^{11} + 25\,770\,076\,T^{12} -$$
$$25\,640\,143\,T^{13} + 18\,116\,522\,T^{14} - 9\,205\,508\,T^{15} + 3\,296\,960\,T^{16} - 773\,888\,T^{17} + 91\,904\,T^{18}\Big)$$

$$-\frac{1}{T^9}\Big(740 - 12\,589\,T + 105\,818\,T^2 - 557\,334\,T^3 + 1\,957\,884\,T^4 - 4\,570\,575\,T^5 +$$
$$6\,803\,506\,T^6 - 5\,866\,266\,T^7 + 2\,123\,368\,T^8 + 42\,684\,T^9 + 2\,123\,368\,T^{10} - 5\,866\,266\,T^{11} +$$
$$6\,803\,506\,T^{12} - 4\,570\,575\,T^{13} + 1\,957\,884\,T^{14} - 557\,334\,T^{15} + 105\,818\,T^{16} - 12\,589\,T^{17} + 740\,T^{18}\Big)$$

$$-\frac{1}{T^{13}}\Big(366 - 6693\,T + 54\,989\,T^2 - 276\,251\,T^3 + 977\,507\,T^4 - 2\,639\,099\,T^5 + 5\,656\,225\,T^6 - 9\,722\,857\,T^7 +$$
$$13\,350\,884\,T^8 - 14\,476\,545\,T^9 + 12\,001\,643\,T^{10} - 6\,918\,985\,T^{11} + 1\,939\,666\,T^{12} + 129\,732\,T^{13} +$$
$$1\,939\,666\,T^{14} - 6\,918\,985\,T^{15} + 12\,001\,643\,T^{16} - 14\,476\,545\,T^{17} + 13\,350\,884\,T^{18} - 9\,722\,857\,T^{19} +$$
$$5\,656\,225\,T^{20} - 2\,639\,099\,T^{21} + 977\,507\,T^{22} - 276\,251\,T^{23} + 54\,989\,T^{24} - 6693\,T^{25} + 366\,T^{26}\Big)$$

$$-\frac{1}{T^9}\big(2084 - 39\,597\,T + 320\,986\,T^2 - 1\,470\,904\,T^3 + 4\,281\,540\,T^4 - 8\,338\,228\,T^5 + 10\,795\,032\,T^6 -$$
$$8\,524\,877\,T^7 + 2\,924\,924\,T^8 + 110\,756\,T^9 + 2\,924\,924\,T^{10} - 8\,524\,877\,T^{11} + 10\,795\,032\,T^{12} -$$
$$8\,338\,228\,T^{13} + 4\,281\,540\,T^{14} - 1\,470\,904\,T^{15} + 320\,986\,T^{16} - 39\,597\,T^{17} + 2084\,T^{18}\big)$$

$$-\frac{1}{T^{13}}\big(150 - 2700\,T + 22\,291\,T^2 - 111\,327\,T^3 + 376\,324\,T^4 - 915\,236\,T^5 + 1\,676\,185\,T^6 - 2\,394\,815\,T^7 +$$
$$2\,678\,620\,T^8 - 2\,255\,714\,T^9 + 1\,354\,314\,T^{10} - 558\,850\,T^{11} + 186\,370\,T^{12} - 99\,184\,T^{13} +$$
$$186\,370\,T^{14} - 558\,850\,T^{15} + 1\,354\,314\,T^{16} - 2\,255\,714\,T^{17} + 2\,678\,620\,T^{18} - 2\,394\,815\,T^{19} +$$
$$1\,676\,185\,T^{20} - 915\,236\,T^{21} + 376\,324\,T^{22} - 111\,327\,T^{23} + 22\,291\,T^{24} - 2700\,T^{25} + 150\,T^{26}\big)$$

$$-\frac{1}{T^9}\big(91\,904 - 854\,304\,T + 3\,966\,144\,T^2 - 11\,918\,062\,T^3 + 25\,050\,910\,T^4 - 37\,618\,117\,T^5 + 39\,735\,922\,T^6 -$$
$$27\,195\,207\,T^7 + 8\,755\,134\,T^8 - 18\,072\,T^9 + 8\,755\,134\,T^{10} - 27\,195\,207\,T^{11} + 39\,735\,922\,T^{12} -$$
$$37\,618\,117\,T^{13} + 25\,050\,910\,T^{14} - 11\,918\,062\,T^{15} + 3\,966\,144\,T^{16} - 854\,304\,T^{17} + 91\,904\,T^{18}\big)$$

$$-\frac{1}{T^{13}}\big(66 - 1147\,T + 9359\,T^2 - 49\,707\,T^3 + 197\,404\,T^4 - 620\,043\,T^5 + 1\,572\,481\,T^6 - 3\,214\,455\,T^7 +$$
$$5\,198\,960\,T^8 - 6\,462\,502\,T^9 + 5\,903\,722\,T^{10} - 3\,637\,752\,T^{11} + 1\,164\,662\,T^{12} - 110\,420\,T^{13} +$$
$$1\,164\,662\,T^{14} - 3\,637\,752\,T^{15} + 5\,903\,722\,T^{16} - 6\,462\,502\,T^{17} + 5\,198\,960\,T^{18} - 3\,214\,455\,T^{19} +$$
$$1\,572\,481\,T^{20} - 620\,043\,T^{21} + 197\,404\,T^{22} - 49\,707\,T^{23} + 9359\,T^{24} - 1147\,T^{25} + 66\,T^{26}\big)$$

$$-\frac{1}{T^9}\big(10\,620 - 146\,319\,T + 860\,304\,T^2 - 2\,856\,598\,T^3 + 5\,924\,190\,T^4 - 7\,861\,443\,T^5 + 6\,530\,144\,T^6 -$$
$$3\,167\,365\,T^7 + 728\,482\,T^8 - 32\,234\,T^9 + 728\,482\,T^{10} - 3\,167\,365\,T^{11} + 6\,530\,144\,T^{12} -$$
$$7\,861\,443\,T^{13} + 5\,924\,190\,T^{14} - 2\,856\,598\,T^{15} + 860\,304\,T^{16} - 146\,319\,T^{17} + 10\,620\,T^{18}\big)$$

$$-\frac{1}{T^{13}}\big(150 - 2700\,T + 23\,471\,T^2 - 128\,293\,T^3 + 489\,649\,T^4 - 1\,380\,312\,T^5 + 2\,989\,455\,T^6 - 5\,102\,435\,T^7 +$$
$$6\,924\,210\,T^8 - 7\,385\,363\,T^9 + 5\,973\,013\,T^{10} - 3\,356\,473\,T^{11} + 973\,872\,T^{12} - 24\,692\,T^{13} +$$
$$973\,872\,T^{14} - 3\,356\,473\,T^{15} + 5\,973\,013\,T^{16} - 7\,385\,363\,T^{17} + 6\,924\,210\,T^{18} - 5\,102\,435\,T^{19} +$$
$$2\,989\,455\,T^{20} - 1\,380\,312\,T^{21} + 489\,649\,T^{22} - 128\,293\,T^{23} + 23\,471\,T^{24} - 2700\,T^{25} + 150\,T^{26}\big)$$

$$-\frac{1}{T^{13}}\big(66 - 1147\,T + 9573\,T^2 - 52\,827\,T^3 + 218\,779\,T^4 - 716\,567\,T^5 + 1\,899\,075\,T^6 - 4\,069\,437\,T^7 +$$
$$6\,924\,304\,T^8 - 9\,074\,616\,T^9 + 8\,723\,721\,T^{10} - 5\,586\,951\,T^{11} + 1\,765\,446\,T^{12} - 67\,042\,T^{13} +$$
$$1\,765\,446\,T^{14} - 5\,586\,951\,T^{15} + 8\,723\,721\,T^{16} - 9\,074\,616\,T^{17} + 6\,924\,304\,T^{18} - 4\,069\,437\,T^{19} +$$
$$1\,899\,075\,T^{20} - 716\,567\,T^{21} + 218\,779\,T^{22} - 52\,827\,T^{23} + 9573\,T^{24} - 1147\,T^{25} + 66\,T^{26}\big)$$

$$-\frac{1}{T^{13}}\big(138 - 2478\,T + 21\,579\,T^2 - 118\,005\,T^3 + 447\,182\,T^4 - 1\,230\,540\,T^5 + 2\,512\,715\,T^6 -$$
$$3\,791\,303\,T^7 + 4\,059\,584\,T^8 - 2\,738\,159\,T^9 + 692\,744\,T^{10} + 437\,538\,T^{11} - 242\,240\,T^{12} - 85\,586\,T^{13} -$$
$$242\,240\,T^{14} + 437\,538\,T^{15} + 692\,744\,T^{16} - 2\,738\,159\,T^{17} + 4\,059\,584\,T^{18} - 3\,791\,303\,T^{19} +$$
$$2\,512\,715\,T^{20} - 1\,230\,540\,T^{21} + 447\,182\,T^{22} - 118\,005\,T^{23} + 21\,579\,T^{24} - 2478\,T^{25} + 138\,T^{26}\big)$$

$$-\frac{1}{T^{13}}\big(138 - 2478\,T + 21\,083\,T^2 - 110\,325\,T^3 + 391\,654\,T^4 - 983\,438\,T^5 + 1\,760\,927\,T^6 - 2\,148\,573\,T^7 +$$
$$1\,444\,820\,T^8 + 260\,535\,T^9 - 1\,703\,464\,T^{10} + 1\,681\,982\,T^{11} - 599\,312\,T^{12} - 15\,422\,T^{13} -$$
$$599\,312\,T^{14} + 1\,681\,982\,T^{15} - 1\,703\,464\,T^{16} + 260\,535\,T^{17} + 1\,444\,820\,T^{18} - 2\,148\,573\,T^{19} +$$
$$1\,760\,927\,T^{20} - 983\,438\,T^{21} + 391\,654\,T^{22} - 110\,325\,T^{23} + 21\,083\,T^{24} - 2478\,T^{25} + 138\,T^{26}\big)$$

$$-\frac{1}{T^{13}}\big(66 - 1147\,T + 9787\,T^2 - 55\,959\,T^3 + 240\,934\,T^4 - 820\,927\,T^5 + 2\,263\,581\,T^6 - 5\,041\,775\,T^7 +$$
$$8\,911\,466\,T^8 - 12\,124\,377\,T^9 + 12\,075\,710\,T^{10} - 7\,945\,190\,T^{11} + 2\,486\,350\,T^{12} + 14\,754\,T^{13} +$$
$$2\,486\,350\,T^{14} - 7\,945\,190\,T^{15} + 12\,075\,710\,T^{16} - 12\,124\,377\,T^{17} + 8\,911\,466\,T^{18} - 5\,041\,775\,T^{19} +$$
$$2\,263\,581\,T^{20} - 820\,927\,T^{21} + 240\,934\,T^{22} - 55\,959\,T^{23} + 9787\,T^{24} - 1147\,T^{25} + 66\,T^{26}\big)$$

$$-\frac{1}{T^{13}}\big(138 - 2478\,T + 22\,163\,T^2 - 126\,515\,T^3 + 507\,605\,T^4 - 1\,498\,010\,T^5 + 3\,322\,935\,T^6 -$$
$$5\,541\,977\,T^7 + 6\,796\,088\,T^8 - 5\,797\,222\,T^9 + 3\,033\,577\,T^{10} - 673\,361\,T^{11} - 15\,386\,T^{12} - 43\,434\,T^{13} -$$
$$15\,386\,T^{14} - 673\,361\,T^{15} + 3\,033\,577\,T^{16} - 5\,797\,222\,T^{17} + 6\,796\,088\,T^{18} - 5\,541\,977\,T^{19} +$$
$$3\,322\,935\,T^{20} - 1\,498\,010\,T^{21} + 507\,605\,T^{22} - 126\,515\,T^{23} + 22\,163\,T^{24} - 2478\,T^{25} + 138\,T^{26}\big)$$

$$-\frac{1}{T^{13}}\big(150 - 3250\,T + 32\,796\,T^2 - 203\,281\,T^3 + 864\,103\,T^4 - 2\,673\,314\,T^5 + 6\,260\,727\,T^6 - 11\,399\,707\,T^7 +$$
$$16\,320\,505\,T^8 - 18\,211\,380\,T^9 + 15\,317\,813\,T^{10} - 8\,895\,347\,T^{11} + 2\,620\,752\,T^{12} - 49\,466\,T^{13} +$$
$$2\,620\,752\,T^{14} - 8\,895\,347\,T^{15} + 15\,317\,813\,T^{16} - 18\,211\,380\,T^{17} + 16\,320\,505\,T^{18} - 11\,399\,707\,T^{19} +$$
$$6\,260\,727\,T^{20} - 2\,673\,314\,T^{21} + 864\,103\,T^{22} - 203\,281\,T^{23} + 32\,796\,T^{24} - 3250\,T^{25} + 150\,T^{26}\big)$$

$$-\frac{1}{T^{13}}\left(66 - 1389\,T + 13\,658\,T^2 - 85\,433\,T^3 + 387\,283\,T^4 - 1\,355\,320\,T^5 + 3\,770\,539\,T^6 - 8\,391\,885\,T^7 + \right.$$
$$14\,754\,069\,T^8 - 19\,926\,600\,T^9 + 19\,716\,005\,T^{10} - 12\,957\,617\,T^{11} + 4\,146\,730\,T^{12} - 128\,412\,T^{13} +$$
$$4\,146\,730\,T^{14} - 12\,957\,617\,T^{15} + 19\,716\,005\,T^{16} - 19\,926\,600\,T^{17} + 14\,754\,069\,T^{18} - 8\,391\,885\,T^{19} +$$
$$\left. 3\,770\,539\,T^{20} - 1\,355\,320\,T^{21} + 387\,283\,T^{22} - 85\,433\,T^{23} + 13\,658\,T^{24} - 1389\,T^{25} + 66\,T^{26}\right)$$

$$-\frac{1}{T^{13}}\left(66 - 1389\,T + 14\,086\,T^2 - 92\,841\,T^3 + 447\,801\,T^4 - 1\,673\,014\,T^5 + 4\,973\,275\,T^6 - 11\,816\,615\,T^7 + \right.$$
$$22\,148\,783\,T^8 - 31\,818\,545\,T^9 + 33\,304\,213\,T^{10} - 22\,846\,169\,T^{11} + 7\,326\,062\,T^{12} + 80\,490\,T^{13} +$$
$$7\,326\,062\,T^{14} - 22\,846\,169\,T^{15} + 33\,304\,213\,T^{16} - 31\,818\,545\,T^{17} + 22\,148\,783\,T^{18} - 11\,816\,615\,T^{19} +$$
$$\left. 4\,973\,275\,T^{20} - 1\,673\,014\,T^{21} + 447\,801\,T^{22} - 92\,841\,T^{23} + 14\,086\,T^{24} - 1389\,T^{25} + 66\,T^{26}\right)$$

$$-\frac{254\,602 - 1\,156\,980\,T + 1\,887\,904\,T^2 - 1\,284\,825\,T^3 + 374\,244\,T^4 - 140\,174\,T^5 + 374\,244\,T^6 - 1\,284\,825\,T^7 + 1\,887\,904\,T^8 - 1\,156\,980\,T^9 + 254\,602\,T^{10}}{T^5}$$

$$-\frac{1}{T^{13}}\left(390 - 7137\,T + 57\,671\,T^2 - 276\,073\,T^3 + 889\,394\,T^4 - 2\,081\,263\,T^5 + 3\,755\,033\,T^6 - 5\,503\,645\,T^7 + \right.$$
$$6\,679\,638\,T^8 - 6\,509\,081\,T^9 + 4\,813\,706\,T^{10} - 2\,450\,176\,T^{11} + 559\,118\,T^{12} + 157\,650\,T^{13} +$$
$$559\,118\,T^{14} - 2\,450\,176\,T^{15} + 4\,813\,706\,T^{16} - 6\,509\,081\,T^{17} + 6\,679\,638\,T^{18} - 5\,503\,645\,T^{19} +$$
$$\left. 3\,755\,033\,T^{20} - 2\,081\,263\,T^{21} + 889\,394\,T^{22} - 276\,073\,T^{23} + 57\,671\,T^{24} - 7137\,T^{25} + 390\,T^{26}\right)$$

$$-\frac{1}{T^{9}}\left(740 - 13\,809\,T + 124\,016\,T^2 - 687\,846\,T^3 + 2\,522\,974\,T^4 - 6\,126\,423\,T^5 + \right.$$
$$9\,455\,952\,T^6 - 8\,420\,271\,T^7 + 3\,169\,924\,T^8 - 38\,730\,T^9 + 3\,169\,924\,T^{10} - 8\,420\,271\,T^{11} +$$
$$\left. 9\,455\,952\,T^{12} - 6\,126\,423\,T^{13} + 2\,522\,974\,T^{14} - 687\,846\,T^{15} + 124\,016\,T^{16} - 13\,809\,T^{17} + 740\,T^{18}\right)$$

$$-\frac{1}{T^{9}}\left(221\,188 - 2\,091\,369\,T + 9\,740\,140\,T^2 - 29\,121\,154\,T^3 + 60\,827\,294\,T^4 - 91\,006\,174\,T^5 + 96\,108\,322\,T^6 - \right.$$
$$65\,935\,315\,T^7 + 21\,289\,360\,T^8 - 54\,376\,T^9 + 21\,289\,360\,T^{10} - 65\,935\,315\,T^{11} + 96\,108\,322\,T^{12} -$$
$$\left. 91\,006\,174\,T^{13} + 60\,827\,294\,T^{14} - 29\,121\,154\,T^{15} + 9\,740\,140\,T^{16} - 2\,091\,369\,T^{17} + 221\,188\,T^{18}\right)$$

$$-\frac{1}{T^{9}}\left(11\,484 - 183\,027\,T + 1\,277\,900\,T^2 - 5\,190\,894\,T^3 + 13\,748\,970\,T^4 - 24\,845\,200\,T^5 + 30\,363\,490\,T^6 - \right.$$
$$23\,110\,831\,T^7 + 7\,980\,852\,T^8 - 93\,064\,T^9 + 7\,980\,852\,T^{10} - 23\,110\,831\,T^{11} + 30\,363\,490\,T^{12} -$$
$$\left. 24\,845\,200\,T^{13} + 13\,748\,970\,T^{14} - 5\,190\,894\,T^{15} + 1\,277\,900\,T^{16} - 183\,027\,T^{17} + 11\,484\,T^{18}\right)$$

$$-\frac{1}{T^{13}}\left(1 - 3\,T + T^2\right)$$
$$\left(138 - 3076\,T + 31\,087\,T^2 - 187\,178\,T^3 + 744\,679\,T^4 - 2\,060\,164\,T^5 + 4\,076\,324\,T^6 - 5\,817\,781\,T^7 + \right.$$
$$5\,885\,413\,T^8 - 4\,035\,562\,T^9 + 1\,688\,939\,T^{10} - 241\,149\,T^{11} - 175\,380\,T^{12} - 241\,149\,T^{13} + 1\,688\,939\,T^{14} -$$
$$4\,035\,562\,T^{15} + 5\,885\,413\,T^{16} - 5\,817\,781\,T^{17} + 4\,076\,324\,T^{18} - 2\,060\,164\,T^{19} +$$
$$\left. 744\,679\,T^{20} - 187\,178\,T^{21} + 31\,087\,T^{22} - 3076\,T^{23} + 138\,T^{24}\right)$$

$$-\frac{1}{T^{9}}\left(3924 - 53\,622\,T + 376\,506\,T^2 - 1\,693\,306\,T^3 + 5\,186\,880\,T^4 - 10\,809\,021\,T^5 + 14\,759\,506\,T^6 - \right.$$
$$12\,048\,049\,T^7 + 4\,288\,748\,T^8 - 11\,088\,T^9 + 4\,288\,748\,T^{10} - 12\,048\,049\,T^{11} + 14\,759\,506\,T^{12} -$$
$$\left. 10\,809\,021\,T^{13} + 5\,186\,880\,T^{14} - 1\,693\,306\,T^{15} + 376\,506\,T^{16} - 53\,622\,T^{17} + 3924\,T^{18}\right)$$

$$-\frac{1}{T^{9}}\left(100 - 690\,T + 1172\,T^2 + 870\,T^3 + 1942\,T^4 - 4488\,T^5 + 2398\,T^6 + 2537\,T^7 - 11\,132\,T^8 + 26\,494\,T^9 - \right.$$
$$\left. 11\,132\,T^{10} + 2537\,T^{11} + 2398\,T^{12} - 4488\,T^{13} + 1942\,T^{14} + 870\,T^{15} + 1172\,T^{16} - 690\,T^{17} + 100\,T^{18}\right)$$

$$-\frac{1}{T^{13}}$$
$$\left(390 - 4277\,T + 19\,479\,T^2 - 47\,991\,T^3 + 67\,946\,T^4 - 49\,150\,T^5 + 24\,551\,T^6 - 68\,739\,T^7 + 85\,748\,T^8 + 56\,192\,T^9 - \right.$$
$$172\,740\,T^{10} + 76\,806\,T^{11} + 67\,096\,T^{12} - 98\,450\,T^{13} + 67\,096\,T^{14} + 76\,806\,T^{15} - 172\,740\,T^{16} + 56\,192\,T^{17} +$$
$$\left. 85\,748\,T^{18} - 68\,739\,T^{19} + 24\,551\,T^{20} - 49\,150\,T^{21} + 67\,946\,T^{22} - 47\,991\,T^{23} + 19\,479\,T^{24} - 4277\,T^{25} + 390\,T^{26}\right)$$

$$-\frac{1}{T^{9}}\left(44 - 585\,T + 4940\,T^2 - 27\,950\,T^3 + 107\,126\,T^4 - 253\,905\,T^5 + \right.$$
$$340\,856\,T^6 - 228\,825\,T^7 + 36\,710\,T^8 + 54\,974\,T^9 + 36\,710\,T^{10} - 228\,825\,T^{11} +$$
$$\left. 340\,856\,T^{12} - 253\,905\,T^{13} + 107\,126\,T^{14} - 27\,950\,T^{15} + 4940\,T^{16} - 585\,T^{17} + 44\,T^{18}\right)$$

$$-\frac{1}{T^{9}}\left(92 - 2268\,T + 21\,636\,T^2 - 102\,276\,T^3 + 257\,966\,T^4 - 306\,446\,T^5 + \right.$$
$$20\,986\,T^6 + 299\,213\,T^7 - 212\,584\,T^8 + 58\,794\,T^9 - 212\,584\,T^{10} + 299\,213\,T^{11} +$$
$$\left. 20\,986\,T^{12} - 306\,446\,T^{13} + 257\,966\,T^{14} - 102\,276\,T^{15} + 21\,636\,T^{16} - 2268\,T^{17} + 92\,T^{18}\right)$$

$$-\frac{562 - 10\,253\,T + 41\,680\,T^2 - 33\,310\,T^3 - 16\,086\,T^4 + 47\,098\,T^5 - 16\,086\,T^6 - 33\,310\,T^7 + 41\,680\,T^8 - 10\,253\,T^9 + 562\,T^{10}}{T^5}$$

$$-\frac{1}{T^{13}}\left(150 - 2150\,T + 14\,238\,T^2 - 56\,239\,T^3 + 145\,675\,T^4 - 260\,253\,T^5 + \right.$$
$$347\,463\,T^6 - 385\,617\,T^7 + 343\,067\,T^8 - 182\,980\,T^9 + 28\,003\,T^{10} - 4749\,T^{11} + 58\,818\,T^{12} -$$
$$79\,184\,T^{13} + 58\,818\,T^{14} - 4749\,T^{15} + 28\,003\,T^{16} - 182\,980\,T^{17} + 343\,067\,T^{18} - 385\,617\,T^{19} +$$
$$\left.347\,463\,T^{20} - 260\,253\,T^{21} + 145\,675\,T^{22} - 56\,239\,T^{23} + 14\,238\,T^{24} - 2150\,T^{25} + 150\,T^{26}\right)$$

$$-\frac{1}{T^{9}}\left(100 - 2440\,T + 24\,213\,T^2 - 128\,041\,T^3 + 406\,481\,T^4 - 835\,664\,T^5 + \right.$$
$$1\,119\,444\,T^6 - 869\,807\,T^7 + 211\,248\,T^8 + 161\,484\,T^9 + 211\,248\,T^{10} - 869\,807\,T^{11} +$$
$$\left.1\,119\,444\,T^{12} - 835\,664\,T^{13} + 406\,481\,T^{14} - 128\,041\,T^{15} + 24\,213\,T^{16} - 2440\,T^{17} + 100\,T^{18}\right)$$

$$-\frac{1}{T^{9}}\left(27\,252 - 186\,588\,T + 621\,624\,T^2 - 1\,315\,084\,T^3 + 2\,116\,734\,T^4 - 2\,601\,462\,T^5 + 2\,258\,726\,T^6 - \right.$$
$$1\,259\,543\,T^7 + 334\,984\,T^8 + 20\,394\,T^9 + 334\,984\,T^{10} - 1\,259\,543\,T^{11} + 2\,258\,726\,T^{12} -$$
$$\left.2\,601\,462\,T^{13} + 2\,116\,734\,T^{14} - 1\,315\,084\,T^{15} + 621\,624\,T^{16} - 186\,588\,T^{17} + 27\,252\,T^{18}\right)$$